Essays on Economic Misallocation

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Dedication

To my loving family: Natalia, Zapote, Sheyla, André and Daniel.

Abstract

This dissertation consists of two chapters. The unifying theme across them is how government policy affects the allocation of resources in an economy, both at the micro and macro level.

The first chapter analyzes the effects of city-level zoning reforms on the spatial distribution of economic activity in a metropolitan area. Using parcel-level property tax and zoning data, I use Minneapolis recent reform, which eliminated single-family zoning lots, to estimate productivity gains in the local housing development sector. I feed the estimated productivity gain into a quantitative spatial model of the Twin Cities, the metropolitan area which Minneapolis is a part of, to compute the effect of the reform on local wages, rents and commuting patterns.

The second chapter, in turn, develops a general equilibrium model with sectoral linkages in which firms face borrowing constraints that can be alleviated by government subsidies. I use it to evaluate how the Brazilian government's policy to direct subsidized credit to specific sectors, called *earmarked* loans, impacts output per worker through two channels. The first one is the general equilibrium effect of alleviating the borrowing constraint of a sector, increasing output. The second channel works in the other direction. In order to raise funds to subsidize loans, the government needs to tax labor and hence distorts households' consumption–labor supply decisions. Whether the first effect dominates the second depends on how relevant the subsidized sector is in the economy's production network structure. I calibrate the model using Brazilian data to study the federal government's decision to increase subsidies for specific sectors in the credit market, perform optimal policy analysis, and investigate how the economy would have performed had the policy not changed.

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Chapter 1

Housing Supply Constraints and the Distribution of Economic Activity: The Case of the Twin Cities

1.1 Introduction

This chapter investigates general equilibrium effects across a metropolitan area when one of its cities implements a zoning reform that allows for more population density. Previous studies have shed light on the potential benefits for cities in the United States to allow more housing development when they face housing supply constraints due to zoning or other regulations. Allowing more development is expected to lower the cost of housing, attracting new residents, which in turn increases the city's workforce and output. What is usually not emphasized by the current literature is that cities in the same urban area will also be affected by the zoning reform. Here, I use theory and data on the recent zoning reform that took place in Minneapolis, MN, to quantify how the distribution of population, workplace, wages and rents rates across Twin Cities metropolitan area are impacted by the reform.

To investigate first and second order effects of a local zoning reform, I develop a quantitative spatial model of a metropolitan area. Since housing in each location are subject to different regulatory constraints, development costs vary by location. In addition, workers are exposed to location preference shocks with respect to where they want to live and work, as well as commuting cost between their residence and workplace locations. I also introduce agglomeration effects and decreasing returns to scale in the production of the consumption good.

One of the main contributions of this chapter is to develop a methodology to infer productivity losses from zoning laws. I model the reform as a shifter in productivity of the housing sector. This interpretation is similar to how one can interpret changes in total factor productivity in business cycle models as changes in capacity utilization. This approach is similar to the ones found in Glaeser et al. (2005) and Herkenhoff et al. (2018), and is based on the empirical evidence such as in Albouy and Ehrlich (2018) that stricter zoning regulations inhibiting development in an area translate to higher housing costs. By allowing more population density, the reform reduces the cost of housing per unit. This is equivalent to a decrease in the marginal cost of producing housing in a location, which drives down rents.

Lower rental rates attract new residents, which move out of other locations in the metropolitan area. When they do so, due to commuting costs, this gives rise to changes in commuting patterns across the metropolitan area after the zoning reform. Not only the city experiences an influx of residents, but its share of workers with respect to the metropolitan area also increases. Since other locations in the metro area lose workers and residents, rents tend to fall in those locations and wages rise, since labor demand is downward-sloping.

I focus on the recent zoning reform that eliminated single-family housing in the city of Minneapolis and its larger impact on the Twin Cities metropolitan area. While Minneapolis is the economic center of the metro area, its population is only about ten percent of the total. Moreover, roughly twenty percent of the jobs in the Twin Cities are located in the city. Until 2019, around seventy percent of the city's residential lots were zoned as single-family units. Starting in 2020, every parcel in the city of Minneapolis admits at least three dwelling units. No the other city inside the metro area thus far has introduced a similar plan for housing reform. Because it took place recently, its effects will take some years to appear in the data. Therefore, the use of a quantitative spatial model is more suited to analyze this problem than more usual microeconometric tools in urban economics.

With the model, I back out housing productivities at the level of the Census tract before the zoning reform. These measures of housing productivity are consistent with rents imputed from data on parcel-level property values. In the model, given land and materials demanded by the housing sector, lower productivity increases the cost of supplying housing. Therefore, rezoning a location by allowing more density is equivalent to increasing the developer's productivity in that same location. Since the reform affected all the locations previously zoned as single-family in Minneapolis, productivity increases in the housing sector in many parts of the city.

To discipline this increase, I combine these measures with zoning data available at the parcel level for Minneapolis. The objective of the exercise is to quantify by how much the productivity of the housing sector changes after the restriction on single-family units is lifted across the city. I project the estimated housing productivity prior to the zoning reform on each tract's share of lots zoned as single-family units, as well as the tract's distance to the city's downtown area. Because these regressors don't vary due to endogenous choices made by the agents in the model and in the data, they are exogenous and thus can be used to inform how housing productivity will change in each location. I find that the median productivity growth is about 9 percent at the tract level.

To quantitatively assess the impact of the zoning reform, I feed the model with the counterfactual productivities and compute the new general equilibrium in the urban area. I find that the upzoning is expected to decrease the cost of housing in Minneapolis by about twenty percent, even with the additional influx of residents from other cities in the metropolitan area. Rents in most other tracts in the metro area also fall, given that part of their local population moves out to Minneapolis. The model also predicts that the policy should attract new residents and workforce to the city by five and two percent, respectively. Most of the new jobs are created in Downtown Minneapolis, which also draws workers from other tracts in the city. Because there are fewer workers in the tracts farther from the city center, downward-sloping labor demand causes wages in those locations rise. Aggregated at the city level, wages in Minneapolis fall by almost one percent.

In addition, the model predicts the second-order effects of the policy coming form the reallocation of the workforce inside the metropolitan area. Workers that move to Minneapolis come from suburban parts of the metropolitan area. Minneapolis is located in the center of the Twin Cities. This implies that the reform generates higher density in the center of the metropolitan area. Higher density is not exclusive to Minneapolis. Other tracts adjacent to Minneapolis also experience population growth. Driving this result is both the increase in wages in these locations outside Minneapolis, as well as the lower costs of commuting costs from these counties to the city.

The results described above highlight the importance of looking at zoning reforms in a broader context outside the city in which it takes place. Because individuals don't need to live and work in the same location, making housing more affordable in one location can have impacts on cities in a commuting distance to it. It also shows how heterogeneous the impacts can be across the metropolitan area.

Many cities in the United States zone most of their residential areas as single-family detached houses. They account for seventy five percent of the residential land in Los Angeles, CA; and seventy nine percent in Chicago, IL, for example. This restriction on development can impact by how much a city can attract new workers, while at the same time driving up housing costs and increasing commuting times from home to work. Many local and state governments have been pushing for zoning reforms that allow more densely packed buildings to increase housing affordability and attract new workers. In recent years, besides Minneapolis, cities such as Seattle and Portland have introduced or passed bills in an effort to reduce or eliminate neighborhoods exclusively zoned as single-family housing. The effects of such policies on neighboring cities is often left out ot the debate. Although the potential gains from upzoning may seem obvious for the city that implements the policy, less clear are the second-order effects that come from population reallocation across an urban area. This chapter contributes to the zoning reform literature by highlighting such general equilibrium effects.

This chapter does not deal directly with the potential distributional conflicts that may arise between renters and homeowners. Zoning rules exist in the real world sometimes for reasons that are not internalized in the model. For instance, homeowners may use zoning to intentionally reduce density around where they live, or to force higher sorting through income in their neighborhoods. They may also want to use their properties as a source of financial investment. In such case, an increase in house prices and rents due to land-use regulations is beneficial to homeowners. My model is silent about these features of the real world, choosing instead to focus on the spatial and labor market implications of the policy.

Related Literature This chapter dialogues with both the literature on quantitative spatial economics and the one on the impact of zoning regulations on spatial misallocation of workers.

The field of quantitative spatial economics has been growing in the past decades, beginning with papers such as Lucas and Rossi–Hansberg (2002) and more recently synthesized in Allen and Arkolakis (2014) and Redding and Rossi-Hansberg (2017). Ahlfeldt et al. (2015) develop a quantitative model of a city building upon international trade models such as Eaton and Kortum (2002). They use the exogenous variation at the city block level of the division and reunification of Berlin to estimate and quantify the agglomeration and dispersion forces present inside a city. Tsivanidis (2020) evaluates the impact of the introduction of a faster public transportation network in a city. Heblich et al. (2020) use data on bilateral commuting flows to inform a quantitative spatial model where commuting costs change due to the introduction of passenger steam railways in 19th century London. These papers highlight the importance of separation between workplace and residence locations for workers inside modern metropolitan areas and the general equilibrium effects of changes in commuting costs across locations. Contrary to these papers, mine focus on changes in housing development costs and their general equilibrium effects.

Owens III et al. (2020) studies the urban structure of Detroit using a model with residential externalities can generate multiple equilibria at the neighborhood level. They include neighborhood-specific fixed costs in housing development to allow for empty neighborhoods in equilibrium when few residents want to live there. Differently from this chapter, their model features a housing cap per neighborhood. While the paper analyzes the interaction between developer incentives and residents location preferences on the distribution of Detroit economic activity, it doesn't focus on zoning reforms. Couture et al. (2019) find that the rise in income among the rich increased demand for luxury amenities in cities, driving housing prices up in downtown areas, pricing out many low-wage workers.

The literature of land-use regulations and economics activity was recently surveyed by Glaeser and Gyourko (2018). At the city level, Kulka (2020) studies the effect of minimum lot sizes on household sorting by income. The paper quantifies the welfare effects of reducing minimum lot sizes using data from Wake County in North Carolina. The paper finds that decreasing minimum lot sizes in rich neighborhoods brings in lower-income workers. Households with at least the area's median income benefit from the policy. This chapter, in contrast, focuses on zoning reforms that change the number of units that can be developed by lot, and considers the effects of the reform on seven counties in the metropolitan area. Parkhomenko (2019) and Khan (2020) study the consequences of decentralized control over land use regulations. Both papers find welfare gains in centralizing land-use regulations in higher levels of government instead of allowing them to be chosen locally. My contribution to this literature is the study of general equilibrium effects of zoning reforms across a metropolitan area.

At the national level, several papers study the role of housing supply constraints in the allocation of economic activity across space. Ganong and Shoag (2017) looks at income convergence across regions in the United States. They introduce nonhomotheticity in housing demand to capture higher housing expenditures among lower income households. They show that increases in housing supply regulations were an important factor to explain why lower wage workers are not moving to high-income places as much as they did three decades ago. Herkenhoff et al. (2018) and Hsieh and Moretti (2019) study land-use regulations and spatial misallocation in the United Sates. Both find negative impacts of land-use regulations on the United States' level of GDP per capita. In particular, Herkenhoff et al. (2018) model land-use regulations in a similar way as this chapter, by interpreting housing productivity heterogeneity as exogenous differences in land use-restriction. Fajgelbaum and Gaubert (2018) study optimal spatial policies in the presence of local agglomeration and congestion forces. They find that spatial sorting by skill and wage inequality in larger cities in the U.S. is too high relative to efficient allocations. Martellini (2020) studies city-size wage premium in the United States and how relaxing housing regulation in cities affect the sorting of workers with different skill levels.

This chapter is organized as follows. Section 1.2 discusses the Twin Cities metro area and the zoning reform implemented in Minneapolis. Section 1.3 presents the spatial model

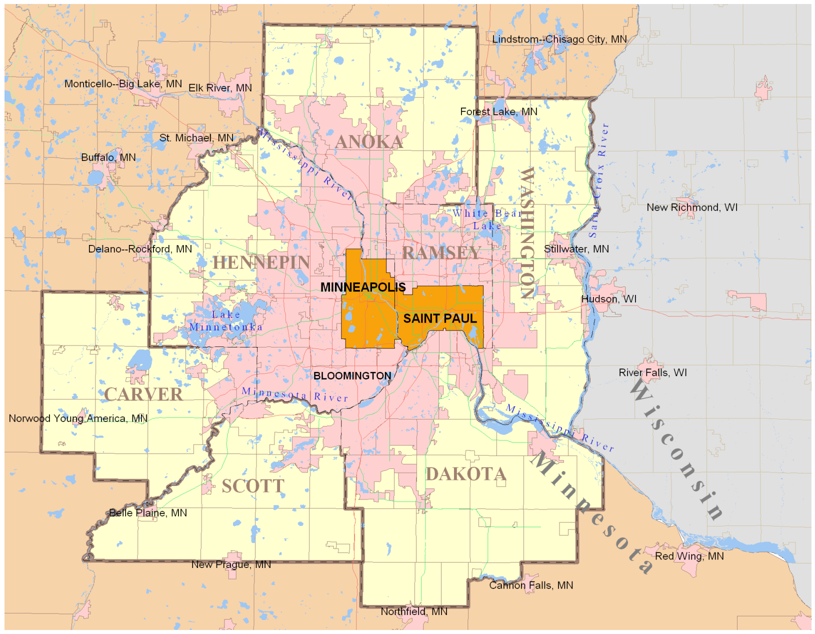


Figure 1.1: Map of Minneapolis, Saint Paul and Bloomington Metropropolitan Statistical Area

of the urban area. Section 2.5 discusses the calibration and estimation strategies used in this chapter. The quantitative counterfactual analysis is presented in Section 1.5. Finally, Section 1.6 concludes.

1.2 The Twin Cities Metropolitan Area

The Minneapolis, Saint Paul and Bloomington Metropolitan Statistical Area, also known as the Twin Cities metropolitan area, is the only MSA in the state of Minnesota. It contains a total of seven counties in the State: Anoka, Carver, Dakota, Hennepin, Ramsey, Scott and Washington. The population of the metro area contains about 3.64 million people, being the third largest population-wise in the Midwest and the 16th largest metropolitan area in the United States.

The Twin Cities metro area gets its name from two neighboring cities that are considered to be the most important in the metropolitan area: Minneapolis and Saint Paul. The former is the largest and most populous city in the state, and the seat of Hennepin County, the state's most populous county. Outside Chicago, Minneapolis is the most densely populated city in the Midwest. The latter is the state's capital and located in Ramsey County, the state's most densely populated county. Figure 1.1 show the map of the Twin Cities metro area, with Minneapolis and St. Paul highlighted.

Even though Minneapolis is economically the most important city in the metropolitan area, it is far from concentrating the majority of its population and labor market. Table 1.1 shows the population and workplace shares in the Twin Cities. Minneapolis population is roughly ten percent of the metro area's population, and twenty one percent of the area's workforce works in the city. In fact, in other counties, at least twenty three percent of their own population work in the same county, highlighting that the economic activity in the metropolitan area is reasonably dispersed. Still, at least ten percent of the workforce living in each county works in Minneapolis, which suggests how important the city is for the overall metropolitan area.

	Population	Workforce	Work in same location	Work in Minneapolis
Anoka	12	7	31	18
Carver	3	2	27	10
Dakota	14	10	37	13
Hennepin*	28	35	57	21
Minneapolis	13	21	45	45
Ramsey	16	19	44	19
Scott	5	2	37	13
Washington	8	4	23	12

* Hennepin considers Hennepin County without Minneapolis

Table 1.1: Commuting Patterns in the Twin Cities (in %)

1.2.1 Zoning Reform in Minneapolis: the 2040 Plan

Until January 1st 2020, about seventy percent of Minneapolis' residential zoning was composed of neighborhoods zoned exclusively for single family detached homes. This meant that each parcel could only have a house where only one family could live in, and the house had to be surrounded by lawn, and not attached to a neighboring house. Figure 1.2 displays in green all the city regions zoned as single-family detached units and in blue all the other strictly residential parcels.

Starting in 2016, Minneapolis City Council proposed a twenty-year comprehensive plan to update the city's long-term plan for itself with respect to its urban landscape, economy and climate impact. The plan, named *Minneapolis 2040*, focuses on a wide variety of topics, such as land use, transportation, housing, public health, arts and culture. Of the interest to this chapter is its plan to change residential zoning in the city, allowing for substantial upzoning.

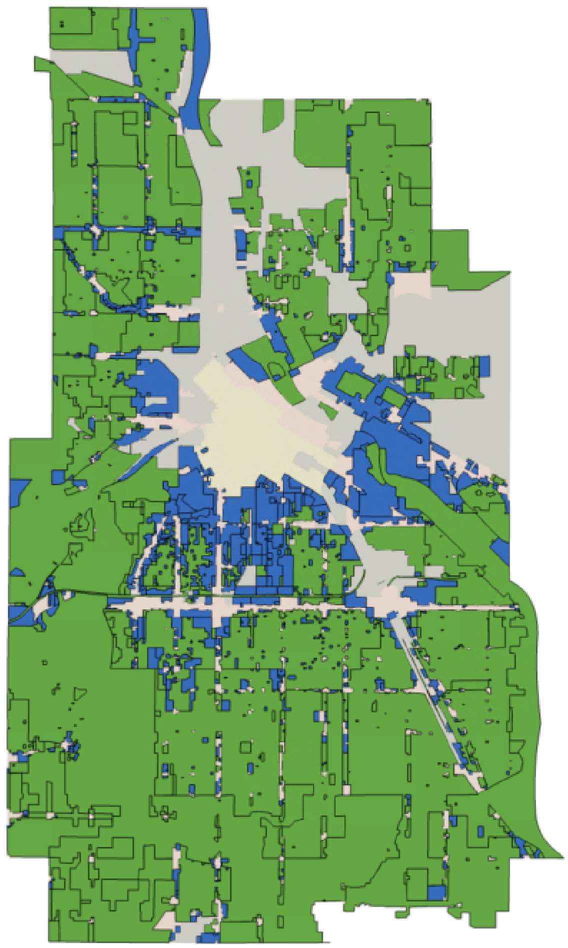


Figure 1.2: Residential Zoning in Minneapolis up to 2019

The plan was approved by the city council and, effective in January 1st, 2020, the city's zoning code changed drastically. Population density in buildings in the downtown area was increased. Along important public transit routes, the city allowed for development of high density units. Nevertheless, the most substantial change regards single-family zoning. All neighborhoods until recently zoned as single family now allow for at most three dwelling units on an individual lot. This has the potential to triple the amount of housing units in most of the city.
An important outcome from this zoning reform will be how the economic activity, population distribution and local labor markets will be affected in the metropolitan area. The reform will not only affect Minneapolis, but all the surrounding cities. It is therefore important to analyze the policy change in the context of the entire metropolitan area, no the city itself. We can expect workers to move in to Minneapolis as a result of an increase in housing affordability, and as a consequence more jobs in Minneapolis and locations nearby. The next section presents an urban model that allows us to make predictions of what to expect in the aftermath of such policy change.

1.3 Model

To quantify the general equilibrium impacts of neighborhood upzoning, I build a quantitative spatial equilibrium model of a metropolitan area. There is a finite and discrete set Ω of neighborhoods. There are four sets of agents: workers, consumption goods producers, housing producers, and absentee land and firm owners. There are $\overline{R}$ workers in the city who can live and work in distinct locations. They are indexed by a pair ij, where $i \in \Omega$ and $j \in \Omega$ correspond to their workplace and residence locations, respectively. Each location produces a homogeneous consumption good, produced by a representative firm. Housing is developed locally as well.

Worker's problem Worker values consumption of a single good, c, housing services, h, exogenous neighborhood amenities, s_j, and idiosyncratic preferences from living in location j and working in i, ϵ_{ij}. I represent commuting costs from j to i by adding a parameter $\kappa_{ij} \geq 1$. I use a Cobb-Douglas utility function to represent the worker's preferences over consumption and housing service. The worker's problem is:

$$\max_{\{c,h,i,j\}} \frac{s_j}{\kappa_{ij}} \left(\frac{c}{\alpha}\right)^{\alpha} \left(\frac{h}{1-\alpha}\right)^{1-\alpha} \epsilon_{ij} \quad \text{subject to} \quad c + r_j h = w_i$$

where w_i is the wage in workplace i and r_j is the pre-tax rental rate of a unit of housing. I assume the worker supplies inelastically one unit of labor. Denote $\hat{V}_{ij} \equiv V_{ij} \times \epsilon_{ij}$ as the counterfactual indirect utility. We can represent it as

$$\hat{V}_{ij} = \frac{w_i s_j}{\kappa_{ij} r_j^{1-\alpha}} \times \epsilon_{ij}.$$

I assume that the worker's preference over local amenities is represented by the vector $\boldsymbol{\epsilon}_k$, which is i.i.d and drawn from a Type II extreme value (Fréchet) distribution:

$$F_{ij}(\epsilon) = \exp\left(-a_{ij}\epsilon^{-\theta}\right),$$

where a_{ij} is the location-specific amenity term, $a_{ij} > 0$ and $\theta > 1$. Worker's location choices to work and live are the ones that maximize their counterfactual indirect utility.

Production in Neighborhood i Technology given by $Y_i = A_i n_i^{\beta}$, $\beta \in (0, 1]$. I introduce agglomeration effects: $A_i = \overline{A}_i n_i^{\eta}$. There is a homogeneous good in the city and the representative firm in each location behaves competitively. Because I allow for decreasing returns to scale in production, potential profits are claimed by absentee firm owners.

Housing Sector in j There's a representative developer that behaves competitively. Production of housing services per location is given by the Cobb-Douglas function $G_j L_j^{\phi} M_j^{1-\phi}$, where L_j is the quantity of land, M_j is materials and G_j is local productivity of the housing sector. The price of materials is given by ι and is homogeneous across locations. Land prices are region-specific, and given by p_j. The developer's problem is given by

$$\max_{L,M} \; r_j G_j L^{\phi} M^{1-\phi} - \iota M - p_j L$$

The price of land is derived from an ad-hoc supply function given by $p_j = (H_j/L_j)^{\overline{\psi}}$, $\overline{\psi} > 0$. Restrictions on development in each neighborhood are interpreted as changes in G_j. Land rents from the housing sector go to absentee land owners.

1.3.1 Equilibrium

Firm's Optimization

Local wages are given by input's marginal productivity: $w_i = \beta \overline{A}_i n_i^{\beta+\eta-1}$.

Worker's Location Choice

Appendix A.3 presents the detailed derivations of the equilibrium conditions of the model. From the law of large numbers, the fraction of workers living in location j and working in neighborhood i, π_{ij}, can be represented by:

$$\pi_{ij} \propto a_{ij} \left(\kappa_{ij} r_j^{(1-\alpha)}\right)^{-\theta} (w_i s_j)^{\theta}.$$

The equation above is a gravity equation for commuting, describing overall patterns of workers' workplace and location choices. It shows that the fraction of the population living in j and working in i is increasing in the location taste shock a_{ij}, wages paid in i, and amenities in j. Similarly, the share of workers is decreasing in costly it is to commute between the pair ij, how high are residential taxes in j, and rent (r_j). Sensitivity to these variables depend on shape parameter θ of location taste

Summing across residential locations, we get the share of workers in location i:

$$\pi_i = \sum_{j' \in \Omega} \pi_{ij'} = \lambda \sum_{j' \in \Omega} a_{ij'} V_{ij'}^{\theta}.$$

The share of workers living in location j is given by:

$$\pi_j = \sum_{i' \in \Omega} \pi_{i'j} = \lambda \sum_{i' \in \Omega} a_{i'j} V_{i'j}^{\theta}.$$

Equivalently, the share of workers living in location j that commute to i to work is given by:

$$\pi_{i|j} = \frac{a_{ij} V_{ij}^{\theta}}{\sum_{i' \in \Omega} a_{i'j} V_{i'j}^{\theta}} = \frac{a_{ij} \left(\frac{w_i}{\kappa_{ij}}\right)^{\theta}}{\sum_{i' \in \Omega} a_{i'j} \left(\frac{w_{i'}}{\kappa_{i'j}}\right)^{\theta}}.$$

Rental Markets

Housing Demand Housing demand for residents in j commuting to i is given by

$$h_{ij} = (1-\alpha)\frac{w_i}{r_j}.$$

Let $\overline{w}_j = \sum_{i \in \Omega} \pi_{i|j} w_i$. Aggregating across working neighborhoods, we get the total housing demand, H_j^d:

$$H_j^d = R_j (1-\alpha)\frac{\overline{w}_j}{r_j}.$$

Housing Supply Using the first-order condition for materials in the housing developer problem, the zero profit condition, and the land supply equation, we get the relationship between housing rent, housing demand and land

$$r_j = \rho_j \left(\frac{H_j}{L_j}\right)^{\psi}, \ \psi \equiv \phi \times \overline{\psi}.$$

Housing Equilibrium From the housing demand equation, the relationship between the number of residents on that neighborhood and total housing demanded is given by

$$H_j = \left[\frac{(1-\alpha)\overline{w}_j}{\rho_j} R_j L_j^{\psi}\right]^{\frac{1}{\psi}}.$$

Labor Market Clearing

In each region, the amount of labor demanded for each skill has to be equal to the amount of labor supplied. The latter is determined by the amount of workers living in each region that commutes to a specific neighborhood to work:

$$n_i = \sum_{j \in \Omega} \pi_{i|j} R_j \quad \forall i \in \Omega.$$

Definition 1 *Given a geography $\{\overline{H}_i\}_{i \in \Omega}$, the equilibrium of the model is defined by a set of location observables such that:*

1. *Given the number of workers in each location, the quantity produced in each region is given by the location's production function.*

$$Y_i = A_i n_i^{\beta}.$$

2. *Given wages, rents and commuting costs, the share of workers commuting from neighborhood j to i follows, $\forall\, i, j \in \Omega$:*

$$\pi_{ij} = \lambda a_{ij} \kappa_{ij}^{-\theta} r_j^{-\theta(1-\alpha)} \left(w_i s_j - r_j \overline{h} \right)^{\theta}.$$

3. *Given wages, number of residents, zoning restrictions and rents, housing supply is given by*

$$H_j = \left[\frac{(1-\alpha)\overline{w}_j}{\rho_j} R_j L_j^{\psi} \right]^{\frac{1}{\psi}}.$$

4. *Given wages, commuting costs, outside-option utility, location preferences, and housing supply, the number of residents in each location follows:*

$$R_j = \sum_{i \in \Omega} \pi_{ij} \overline{R}, \quad \forall j \in \Omega.$$

5. *Given wages, zoning restrictions and fixed costs, rents are given by*

$$r_j = \rho_j \left(\frac{H_j}{L_j} \right)^{\psi}.$$

6. *Given the number of residents in each neighborhood and commuting probabilities, the labor supply in each neighborhood is given by*

$$n_i = \sum_{j \in \Omega} \pi_{ij} \overline{R}.$$

7. *Given the number of workers in each location, local output and prices, firms' first-order conditions determine the wages.*

$$w_i = \beta \overline{A}_i n_i^{\beta+\eta-1}.$$

1.3.2 The Effects of Changing Zoning Regulations

In this model, changes in local zoning regulations are interpreted as changes in local productivity of the housing sector, G_j. Therefore, if a neighborhood is allowed to build more housing units per parcel or decreases the minimum lot size of each parcel, the model captures these changes as increases in G_j.

In the model, the mechanism works as follows. When housing productivity goes up in a location, housing can be produced at lower marginal cost. This has the effect of lowering rents for those already residing in location, which is equivalent to a movement along the housing demand curve. As a consequence, residents already living in the location demand more housing. Residential amenities may move upwards or downwards, depending on how much rent and housing demand respond to the change.

The second-order effects of the policy change come from the general equilibrium structure of the model. Due to lower rents in the location, residents from other locations move, which is equivalent to a shift in the housing demand curve. As an effect, rents goes up. The population and rent increases unequivocally increases taxes collected in the location, making room for a higher supply of neighborhood amenities, which again reinforces the incentives to move in. Because of commuting costs, some of the new residents change their workplace location to work nearby. The possibility of a downward-sloping labor demand curve if $\beta + \eta < 1$, wages tend to fall locally and rise in locations farther away that lost residents and workers.

Other general equilibrium effects are also present. For instance, locations that lose workers producing the consumption good due to the spatial reallocation of residents will observe an increase in local wages, an unintended effect of the policy. In addition, because of commuting costs, locations close to the one which implemented the policy may observe an increase in population as well. These results highlight the importance of analyzing changes in housing policies in a broader context other than the city or county that implemented them if there are nearby regions that will be directly impacted by it.

1.4 Data and Calibration

In this section, I apply the model presented above to analyze the impact of allowing for upzoning in the Minneapolis 2040 plan. I set up the model so that it replicates patterns of the data on the Twin Cities before January 1st 2020, when the new zoning rule took place. That is, the model is supposed to replicate the commuting patterns, local population and labor force, rents and wages across the Twin Cities metro area when most of the residential part of Minneapolis was zoned as single-family, detached, units. I then use the data on higher-density areas to inform the change in housing productivity we should expect to happen when the neighborhoods are allowed to upzone.

The seven counties comprising the Twin Cities metro area contain 702 census tracts in total. Of these tracts, 113 are in the city of Minneapolis. The objective is to use data at the tract level on housing, population, wages, commuting patterns, property taxes, rents and commuting costs to inform the model.

1.4.1 Mapping to Data

The main data sources used for the empirical exercise are the following. I use data on wages, residents and workers from the Longitudinal Origin-Destination Employment Statistics (LODES). They provide origin and destination data on the population of workers that at the Census block level, as well as data on wages by brackets. I use Minnesota Geospatial Commons' Metro Regional Parcel Dataset, which compiles parcel-level data for all the seven counties, as the source for zoning and rents in each Census tract. Monthly rent is calculated using the following formula:

$$\frac{r}{1+r}\frac{\text{Average Building Value/Units}}{1-(1+r)^{-T}}, \quad r = 0.06/12,\ T = 20 \times 12$$

Commuting costs are calculated using IRS estimate of \$0.58 cents per mile. I compute distance across locations in miles using Google Maps. I use local, county-level property tax rates to calibrate τ_j. Productivity can be obtained by inverting the model to match wages.

1.4.2 Gravity Equation

Following Monte, Redding and Rossi-Hansberg (2016), I regress commuting patterns on commuting costs, origin and destination fixed effects to identify the shape parameter of the Frechet distribution, θ:

$$\log\left(\frac{\pi_{ij}}{\pi_{jj}}\right) = -\theta \log\left(\frac{\kappa_{ij}}{\kappa_{jj}}\right) + \mu_i + \mu_j + u_{ij}$$

The estimated θ was 4.4, within the bounds of the literature. For location taste shock, I use $a_{ij} = \pi_{ij}\left(\frac{\kappa_{ij}}{w_i}\right)^{\theta}$.

Identification of Housing Productivity Parameters

From the equilibrium conditions of the model, I can write the housing productivity parameter for every tract as proportional to MSA-wide parameters from the model, rent, population density (residents per square mile) and the data equivalent of $\overline{w}_j$:

$$G_j = \text{constant} \times \text{Rent}_j^{-1}\left\{\left(\frac{\text{Residents}_j}{\text{Land}_j}\right)\left[(1-\alpha)\frac{\text{Average Wage}_j}{\text{Rent}_j}\right]\right\}^{\psi}.$$

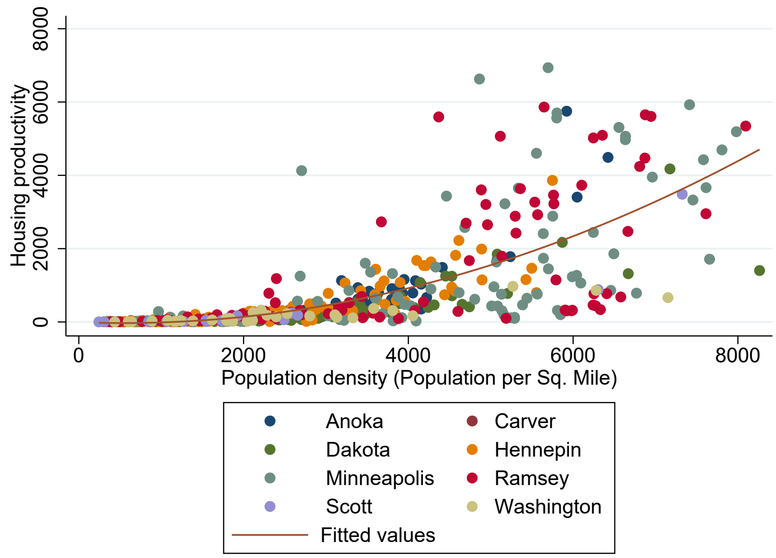


Figure 1.3: Relationship Between Model-Based Housing Productivity and Population Density

Given parameters and data, I can recover G_j. Heuristically, given rents and square mileage, a higher number of residents imply that housing can be built at a lower marginal cost, suggesting higher productivity of the housing sector. Similarly, given wages and population density, a higher value for rent implies that housing is developed at a higher marginal cost, which suggests lower productivity of the housing sector in the particular tract. As Figure 1.3 shows, the model-inferred productivities are associated with higher density in each Census tract, in line with what is expected in this framework.

In addition, pinning down G_j is crucial for the model to reproduce rents as seen in data. As Figure 1.4 shows, if I assume that G_j is equal to one in every tract, the model does a much worse job at matching rent rates at the tract level compared to the specification in which I allow tract-level productivity differences for the housing sector.

Identification of Amenities

From the location choice problem faced by the workers, we have the following system of equations. For every $j \in \Omega$:

$$\pi_j = \lambda \sum_{i \in \Omega} a_{ij} \left(\kappa_{ij} r_j^{(1-\alpha)} \right)^{-\theta} (w_i s_j)^{\theta}.$$

From the data and estimation of θ, we know $\pi_j, a_{ij}, \kappa_{ij}, w_i, r_j \ \forall i, j \in \Omega$. The objective is to find the set of s_j that solves the system above. Note that the right-hand side of the equation is homogeneous of degree zero with respect to the vector of amenities, since

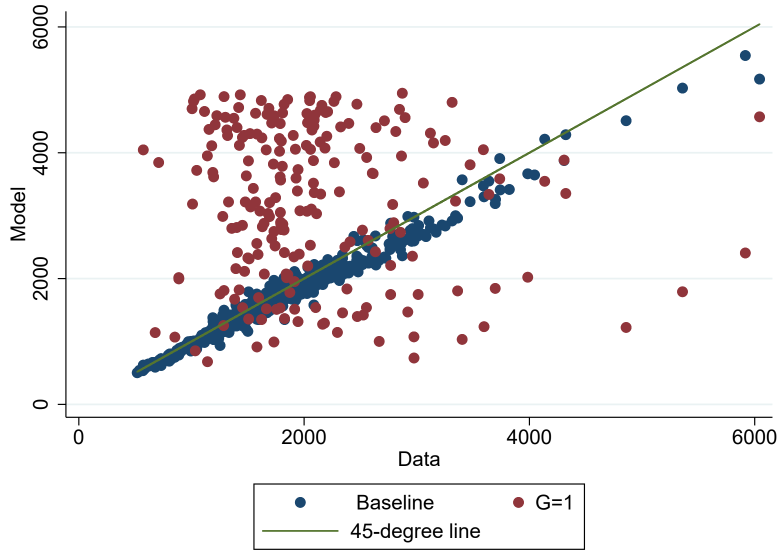


Figure 1.4: Model Fit–Baseline vs. Homogeneous Productivity in Housing Sector

$\lambda^{-1} = \sum_{i,j\in\Omega} a_{ij} \left(\kappa_{ij} r_j^{(1-\alpha)}\right)^{-\theta} (w_i s_j)^{\theta}$. Therefore, it is sufficient to normalize λ to one and solve for s_j such that it matches the distribution of residents in each Census tract:

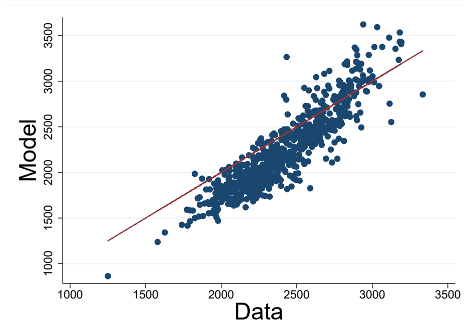
$$s_j = \frac{\pi_j r_j^{\theta(1-\alpha)}}{\sum_{i\in j} a_{ij} (\kappa_{ij})^{-\theta} (w_i)^{\theta}}.$$

1.4.3 Calibration of MSA-Wide Parameters

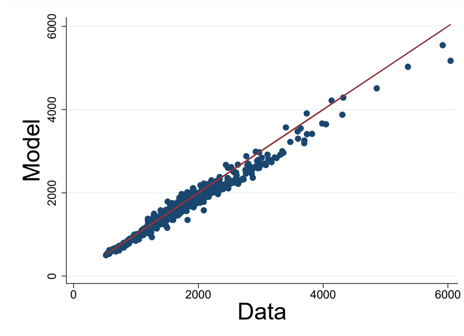
The values I pick for the parameters on the worker and consumption goods production side come from standard sources in the literature. The value of the Cobb-Douglas parameter α is set to 0.76, as in Davis and Ortalo-Magne (2011). The value for β comes from Ahlfeldt et al. (2015) and is set to 0.8, while η is set to 0.06 as in Ciccone and Hall (1996). The parameters $\overline{\psi}$ and ϕ governing the local housing development come from Severen (2018). Table A.1 in Appendix A.2 summarizes the parameters used in the model.

1.5 Quantitative Exercise: The Impact of Zoning Reform

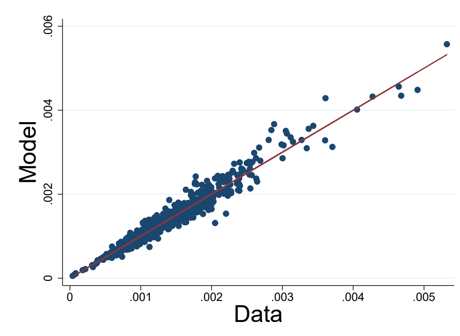
Using the estimated model, I evaluate the impact of Minneapolis zoning reform on the equilibrium prices and allocations across the metropolitan area. In my model, a zoning reform allowing for more density is interpreted as an increase in G_j. This modeling choice assumes that allowing for the development of more housing units in a single plot of land reduces the cost per unit. Appendix A.1 provides a microfoundation as to why a zoning reform allowing for more units can be interpreted as an increase in tract-level productivity.



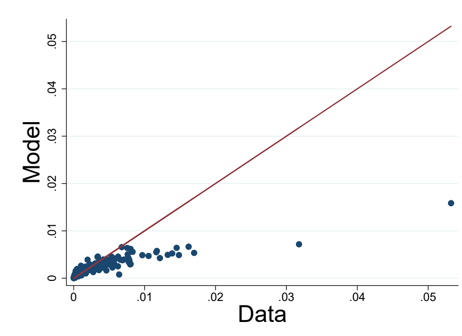
(a) Wages



(b) Rents



(c) Population Share



(d) Worker Share

Figure 1.5: Model Fit
Scatter Plots and Trend Lines

The model is able to replicate the data well, as Figure 1.5 shows. In particular, the model can match very well the distribution of rent values and population shares across the Twin Cities metropolitan area. The model can also successfully replicate the wage dispersion as seen in the data. Where the model does not do as well is at matching the upper end of the distribution of the share of workers working at each location. Figure 1.5d shows that the model underpredicts the share of workers in tracts where they are more highly concentrated.

1.5.1 Zoning Reform in the Model

In this section, I discuss how I incorporate the zoning reform on G_j. The *Minneapolis 2040* plan rezoned the parcels previously marked as single family to allow for up to three dwelling units. As shown in Figure 1.2, single-family units constituted the vast majority of the residential parcels in the city. In my quantitative exercise, this reform is interpreted as an increase in G_j for the locations j that are affected by the reform.
To perform the exercise, I match the Census tracts with Minneapolis zoning map prior to the housing reform. The mapping between Census tracts and municipal zoning is not one-to-one, which means that zoning is not homogeneous for each tract. Restricting attention

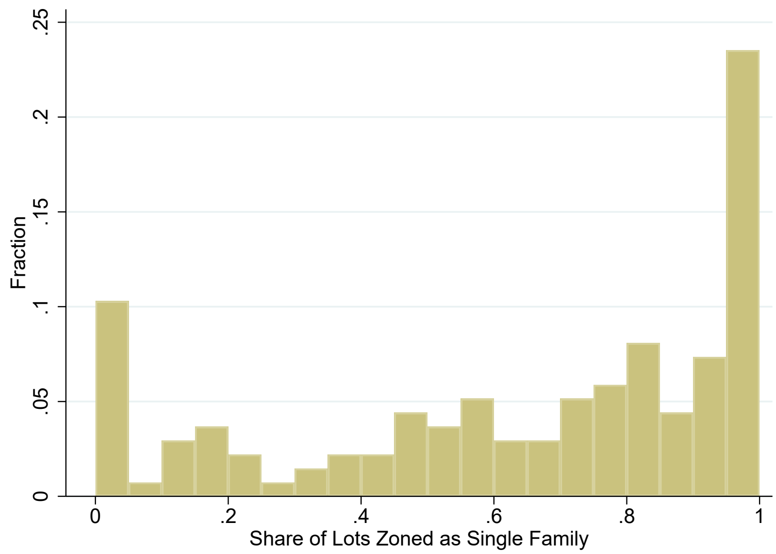


Figure 1.6: Distribution of Tracts by Share of Lots Zoned as Single Family

to residential zoning, about ten percent were fully zoned as single family. Figure 1.6 plots the histogram of the distribution of tracts by share of lots zoned as single family.
I then implement the following empirical strategy: with the estimated housing productivities G_j, I run the following regression:

$$G_j = \beta_0 + \beta_1 \times \text{share of single-family units}_j + \beta_2 \times dist_j + \beta_3 \times dist_j^2 + u_j,$$

where share of single-family units$_j$ represents the tract's share of residential parcels previously zoned as single family, and $dist_j$ is the tract j's distance, in miles, to the most central tract in Downtown Minneapolis. When I run this regression, the coefficient for share of single-family units and distance squared are negative. The fact that the former is negative serves as validation of the model.
I discipline the G_j's after policy in the following way: I use estimated equation above and set the share of single-family units to the minimum that I observe in the data, which is 0.03%. Because $\beta_1 < 0$ and different tracts have different shares of parcels zoned as single family, housing productivity will increase more in the tracts with higher initial shares of single-family units. Moreover, since the share comes from the zoning code, not de facto development, this method ensures that the productivity increase I introduce in the model primitives are exogenous. In addition, the inclusion of the quadratic polynomial form for distance allows my counterfactual housing productivities to increase more in tracts that are closer to Downtown Minneapolis. Adding this quadratic form in the construction of the counterfactual assumes that building at a lower marginal cost is likely increase in places that are already close to dense locations. Using the methodology, I find that median productivity

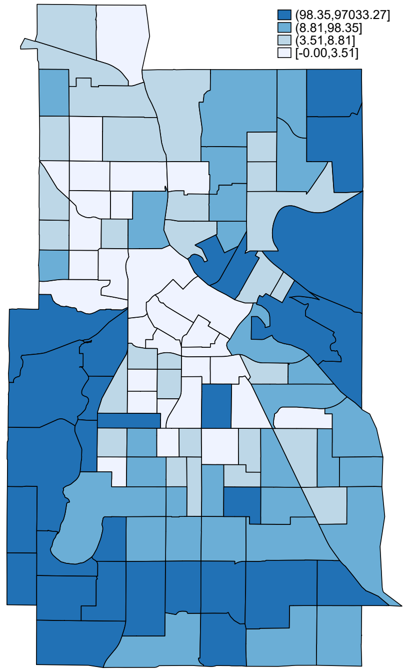


Figure 1.7: Distribution of Housing Productivity Growth in Minneapolis Before and After The Zoning Reform

in the housing development sector is expected to grow by 9 percent in the city. Still, productivity grows much more in some tracts, particularly the ones in the southwest region of the city, which is populated by big single-family homes. Figure 1.7 shows the change in the distribution of housing productivities inside Minneapolis.

1.5.2 Results

I now present the results of the quantitative exercise where I interpret the zoning reform in Minneapolis as a change in the productivity of the housing development sector. Figure 1.8 shows the impact of upzoning on rents. Overall, rent falls about 25 percent in Minneapolis. At the same time, population increases by about 4.6 percent in the city, as shown in Figure 1.9. Due to population reallocation, rents fall in other locations. The effects of the housing reform on rents is heterogeneous across tracts. As Figure 1.10 shows, this is true even for tracts outside Minneapolis. This highlights the importance of taking these general equilibrium effects across the metropolitan area: a decrease in marginal cost of producing housing in Minneapolis attracts workers to the city. This affects these workers' workplace decision, which consequently affects wages. In turn, wages and rents in other locations also respond to this migration decision, which further generates other migration decisions in other locations in the metropolitan area as well.

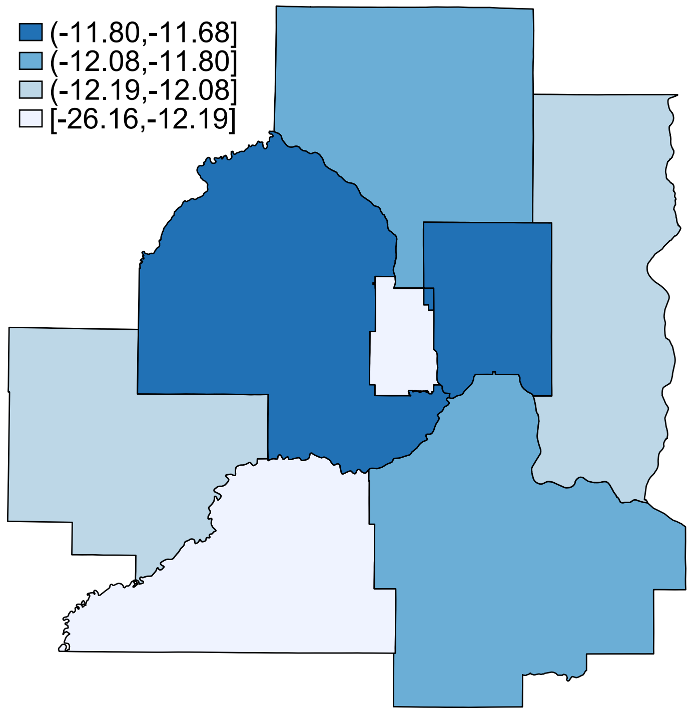


Figure 1.8: Impact on Rents

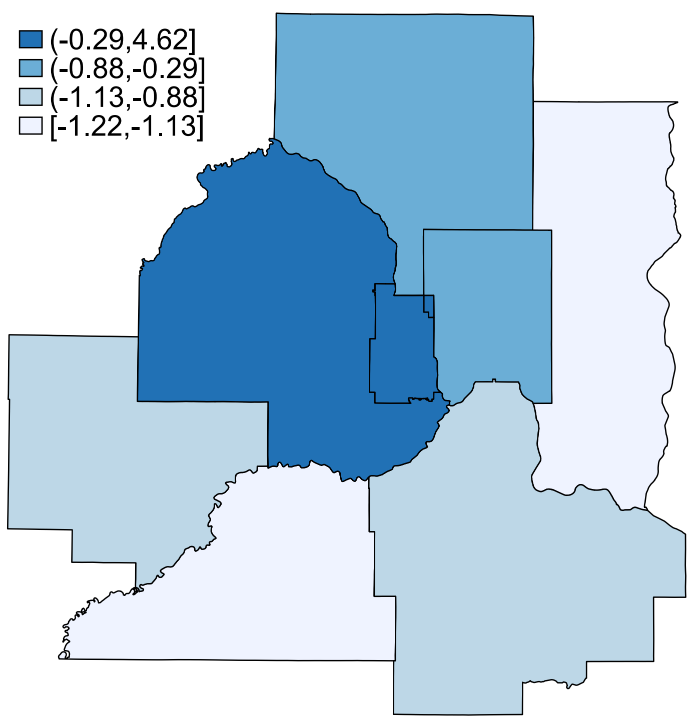


Figure 1.9: Impact on Distribution of Population

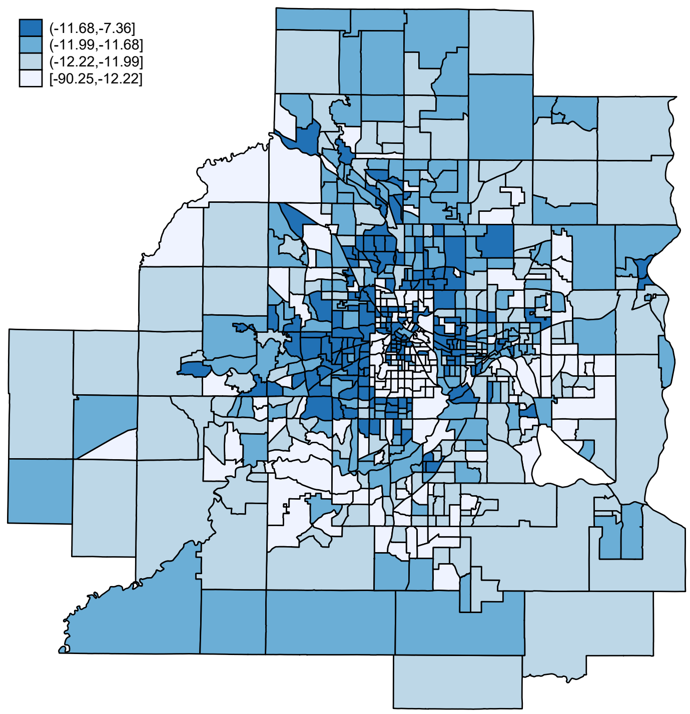


Figure 1.10: Impact on Rents–Tract Level

Inside Minneapolis, the change in rents is negatively correlated with the population changes, as reflected in Figure 1.11. Many tracts experience a large inflow of residents, but still experience a drop in rents. As Figure 1.12 suggests, the mechanism explaining this large influx couples with rent drops in many tracts is largely explained by the negative association between the variation in rents and the tract's pre-policy share of lots zoned as single-family units. Intuitively, since these tracts are now allowed to develop more housing units per lot due to the zoning reform, they can supply more housing to residents at a substantially lower cost. Finally, the drop in rents is also negatively associated with the tract's distance to Downtown Minneapolis.

Upzoning in Minneapolis also has effects on wages across the metropolitan area. Figure 1.14 presents the predicted wage changes from the baseline resulting from the upzoning. At the aggregate level, wages rise in all counties, as well as in Minneapolis. Wages rise the most in Hennepin County, where they rise by about 0.42 percent. As Figure 1.15 shows, the wage increases outside Minneapolis are explained by a significant drop in the number of agents working in those locations.

Looking at the results at the tract level allows us to inspect the mechanism more carefully. As Figure 1.16 shows, while wages increase in most tracts, many of them experience a wage drop. Figure 1.17 shows the reallocation of workers across the metro area. In addition, Figure 1.18 shows that the model implies that workers reallocate outside of Downtown

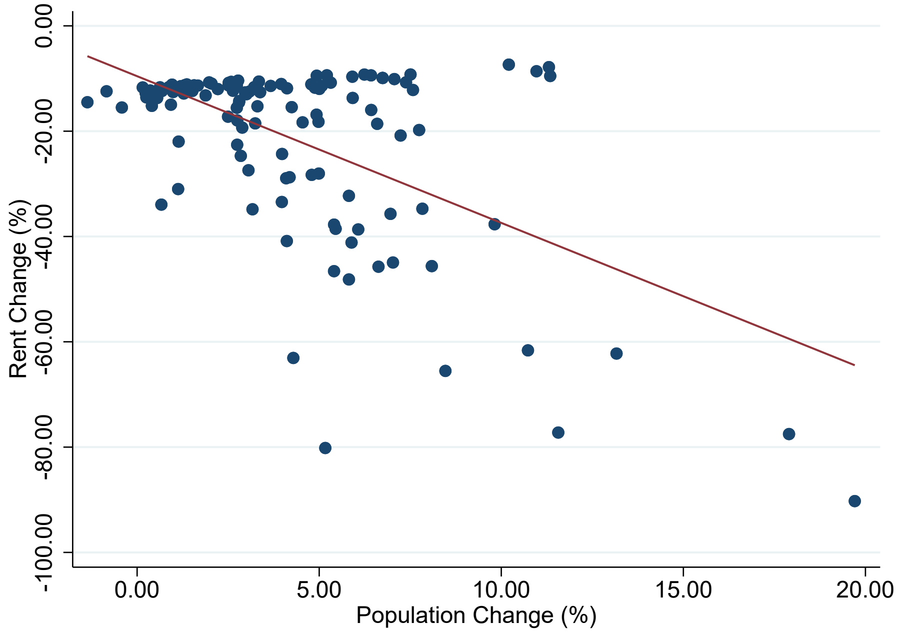


Figure 1.11: Relationship Between Population and Rent Changes
Scatter Plot and Trend Line

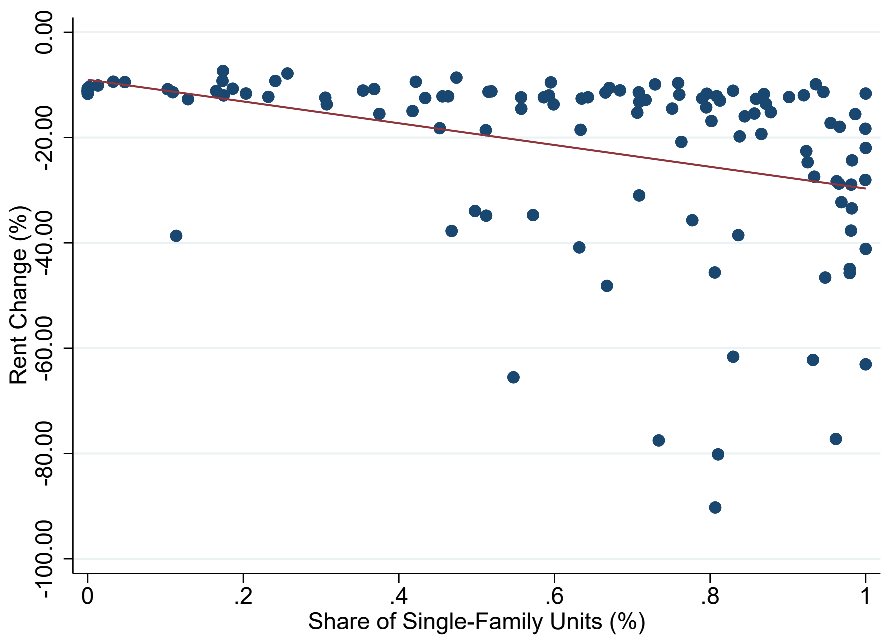


Figure 1.12: Relationship Between Rent Change and Share of Single-Family Units
Scatter Plot and Trend Line

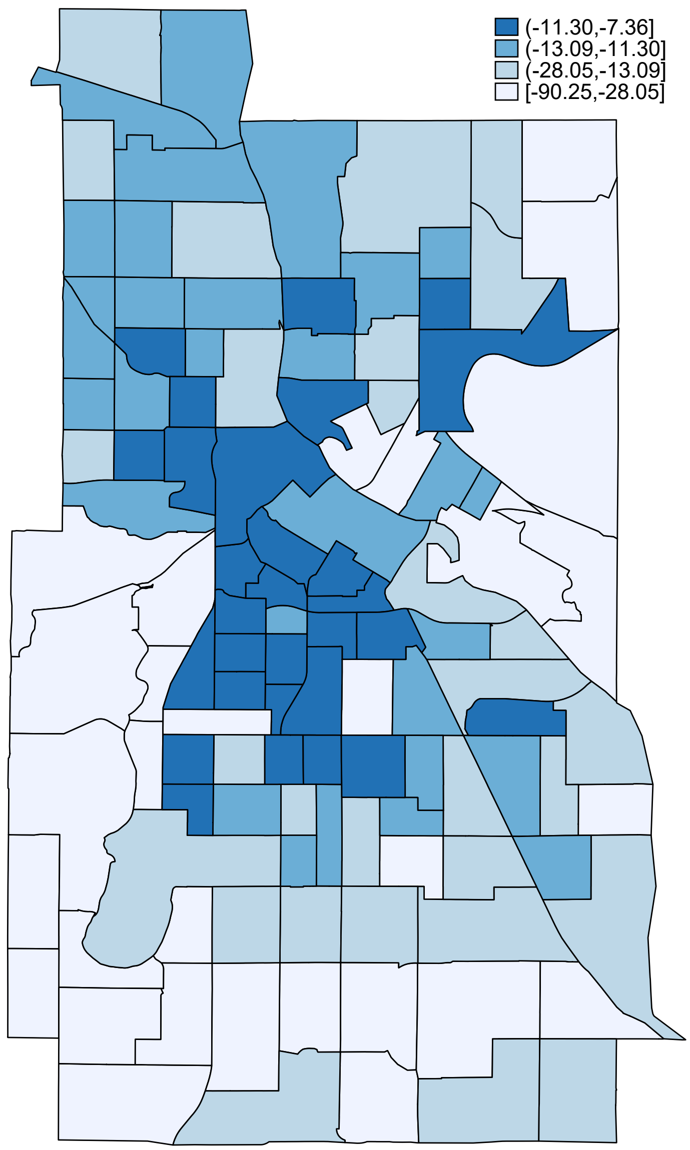


Figure 1.13: Effect of Housing Reform on Rents: Minneapolis

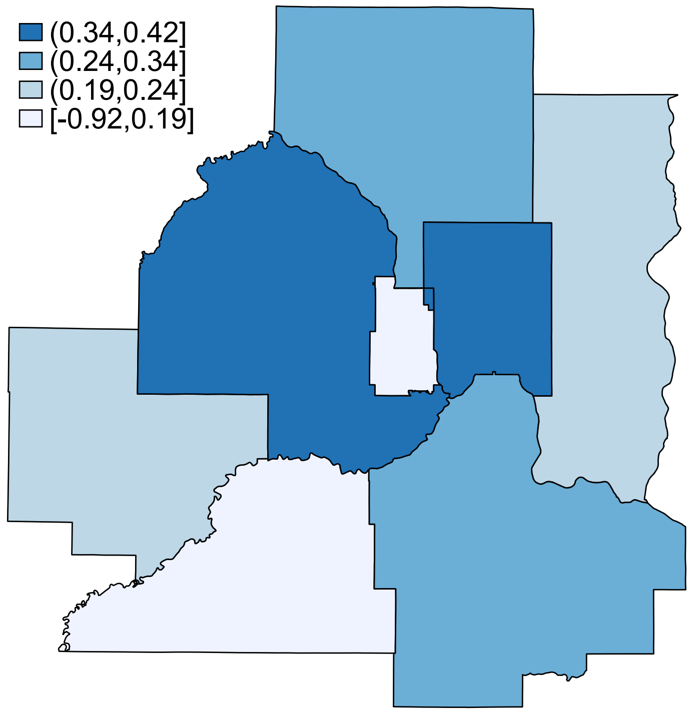


Figure 1.14: Impact on Wages

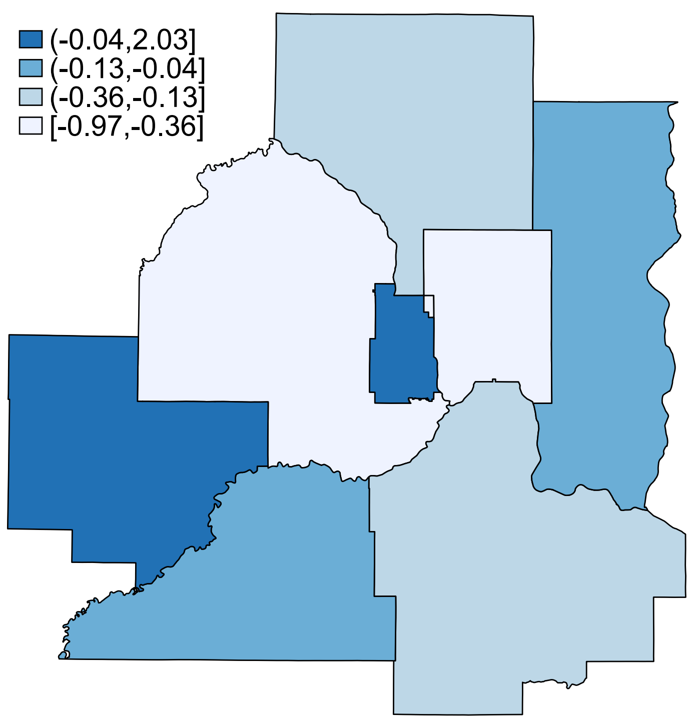


Figure 1.15: Impact on Distribution of Workers

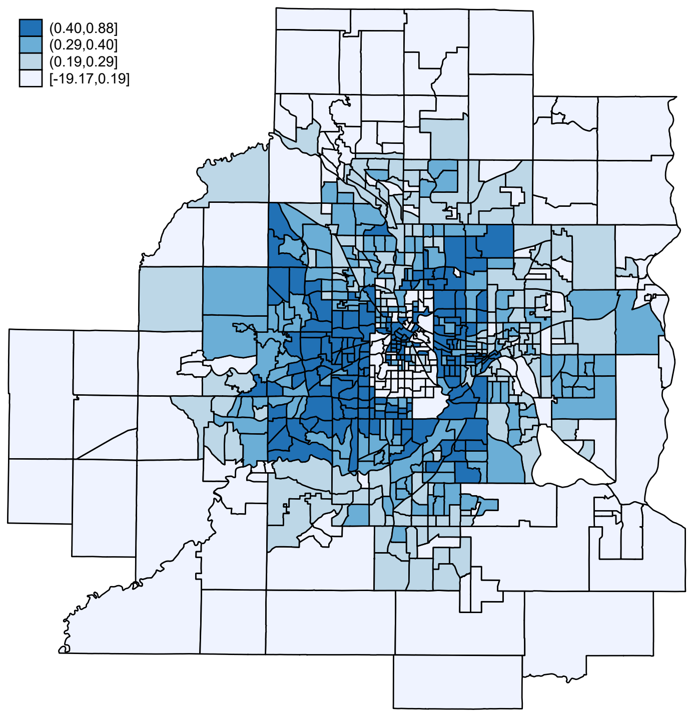


Figure 1.16: Impact on Wages: Tract Level

Minneapolis, located in the center of the city, to tracts in the southwest and northeast parts of the city. Two forces are at play here. First, the parameters for the production function show that labor demand is downward sloping. Therefore, less workers in Downtown Minneapolis implies higher wages for the workers that keep working in the city center, even in the presence of agglomeration effects. Finally, because many agents decide to move to Minneapolis to reside in the areas mostly affected by to the zoning reform, as shown in Figure 1.19, many decide to work close by since commuting costs are lower. Further, the estimated set of location pair preferences from the Frechet distribution, a_{ij}, capture a strong preference for workers who live in Minneapolis to work around where they live.

Finally, we can look at the effects of the housing reform on productivity across the Twin Cities. Figure 1.20 plots the changes in productivity as a result of worker reallocation. The majority of the productivity gains occur in the tracts outside of downtown Minneapolis, which is highlighted in Figure 1.21. Driving this result is the increase in workers in those locations, which induces agglomeration effects that increase productivity in those tracts.

1.6 Conclusion

In this chapter, I analyzed the general equilibrium effects of upzoning in a city from the perspective of the metropolitan area. I built a spatial model with heterogeneous locations,

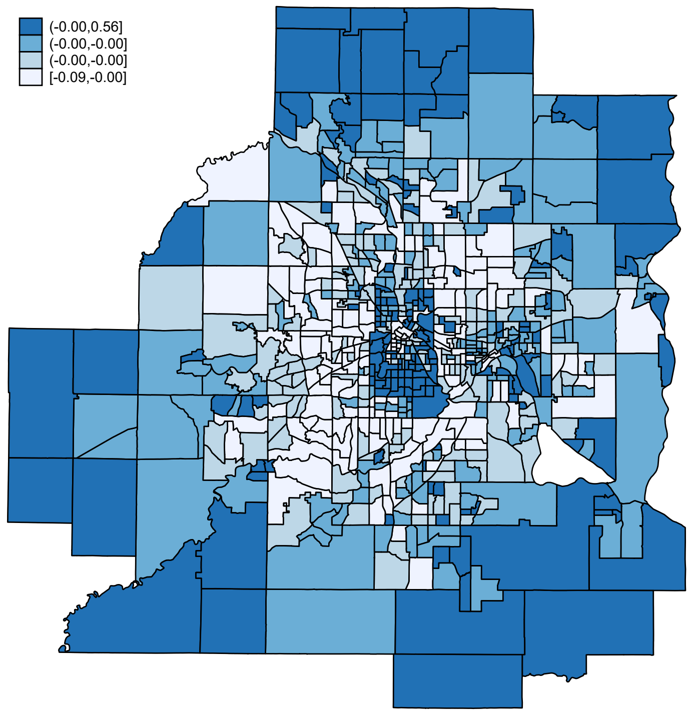


Figure 1.17: Impact on Distribution of Workers: Tract Level

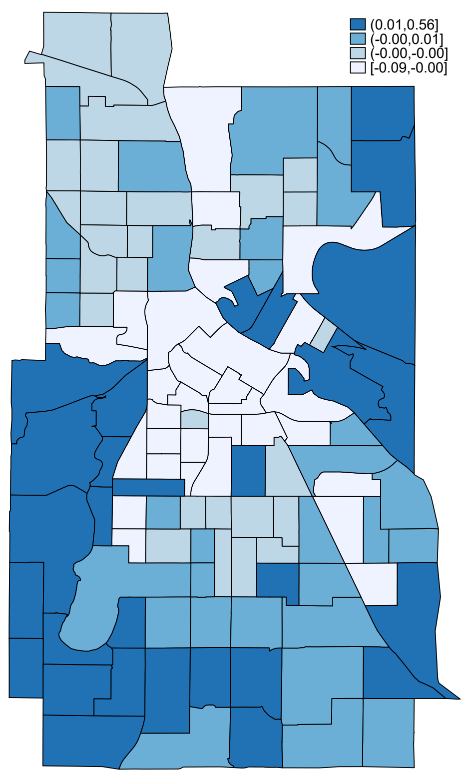


Figure 1.18: Distribution of Workers: Tract Level (Minneapolis)

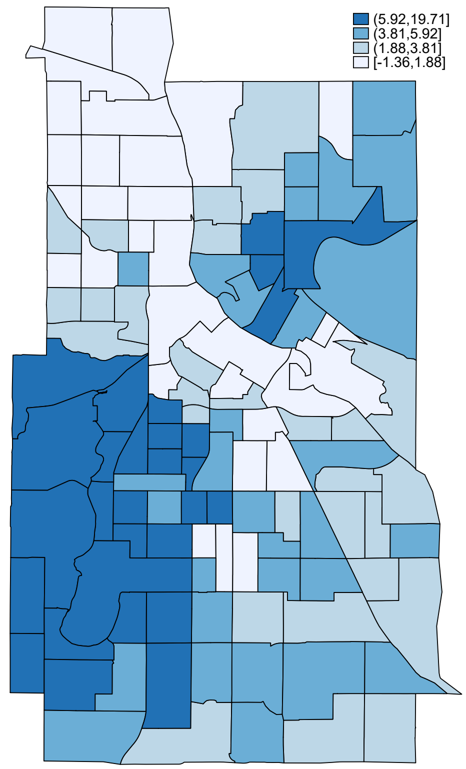


Figure 1.19: Impact on Distribution of Population: Tract Level (Minneapolis)

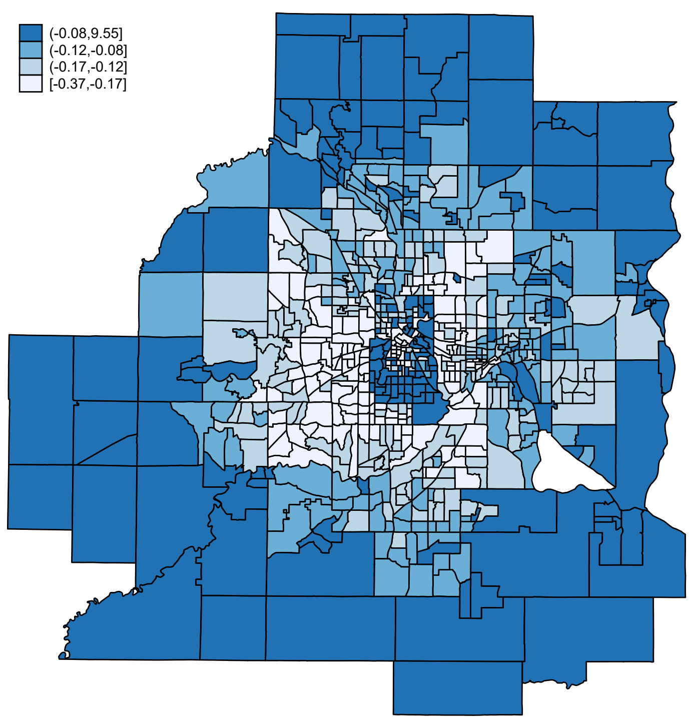


Figure 1.20: Impact on Productivity: Tract Level

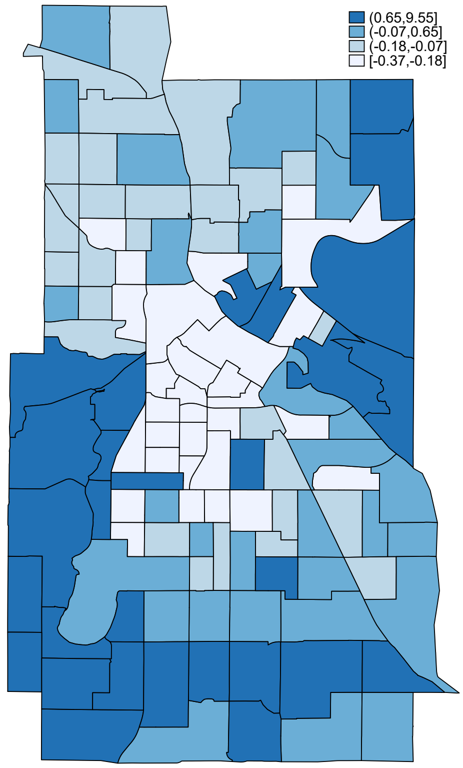


Figure 1.21: Impact on Productivity: Tract Level (Minneapolis)

amenities, productivities, agglomeration and congestion forces, and commuting to quantify the impact of the policy. Local housing policies affect the equilibrium outcomes not only of the city that implements the aforementioned policy, but also of the ones directly connected to it in the greater urban area.

I find quantitatively important effects throughout the metropolitan area. Housing becomes more affordable in the desired location, but this effect also spills over most of other counties as well. At the same time, I find that upzoning is likely to attract more workers, but at the cost of lowering wages due the increase in labor supply locally. In general, the whole metropolitan area benefits from the policy, not just the city which implemented the policy. The results from this chapter highlight the importance of analyzing housing reforms in the perspective of a larger metropolitan area. They may have unexpected benefits and losses to nearby cities that could potentially be taken into account when discussing housing policies.

Chapter 2

Earmarked Loans and Economic Performance in Brazil

2.1 Introduction

This chapter develops a multi-sector model to understand the impact of government credit subsidy policies directed towards specific sectors on output per worker in the presence of a distortionary labor tax, using the Brazilian economy's recent experience with such policy as its quantitative experiment. The model allows for sectoral linkages in production and imposes working capital constraints in the firms' maximization problem. We allow for heterogeneous input demands in the network structure, which implies that some sectors impact others, and thus aggregate output, differently. Therefore, marginally relaxing different sectors' borrowing constraint can generate different impacts on aggregate output due to general equilibrium effects, which implies that any subsidy should take these effects into account.

Another consequence of the credit subsidy policy is the distortion in consumption-labor decisions by households. Because the government needs to tax labor income to subsidize interest rates on loans, any decision to increase subsidies to a specific sector involves a tradeoff.

Since 2010, Brazil has implemented a series of policies to foster economic growth. To achieve this goal, the government heavily used three policy instruments: i) price freezes of "strategic" inputs whose prices are regulated, such as the price of electricity and gasoline; ii) tax exemptions for some sectors, in particular manufacturing industries; and iii) directed, subsidized lending to firms and households. However, as Figure 2.1 shows, the drop in TFP since 2010 has been sharp and consistent. This suggests that these policies may have contributed negatively to the country's economic performance. For instance, after growing

7.6% in 2010, output growth slowed down to an average rate of 2.2% from 2011 to 2014.

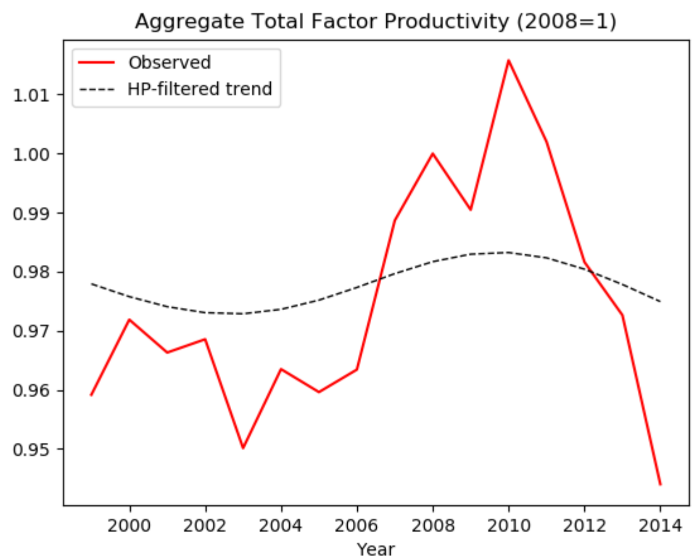


Figure 2.1: Total Factor Productivity–Observed and Trend (2008=1)

In this chapter, we narrow down our analysis to the third instrument. Directed, subsidized lending to specific sectors or household demographics is formally called earmarked lending. One of the main sources of earmarked credit to firms is the Brazilian Social and Economic Development Bank (BNDES hereafter). By earmarking credit, the government generates substantial differences in borrowing costs across firms and sectors. Therefore, this credit policy change in Brazil potentially had non-negligible effect on the economy. By varying how much credit to subsidize and which sectors to focus the policy on, financial frictions were exacerbated in some sectors and remedied in others.

To implement such a large-scale subsidy policy, the government has to generate revenues. In Brazil, this is done mainly through a combination of labor income taxes and government borrowing. This means that even if alleviating firms' financial constraints has the beneficial outcome of increasing output, the distortions arising from taxation can potentially counterbalance this effect, making such policy undesirable.

The way we chose to think about this policy is by looking at how financial constraints affect an economy with sectoral linkages. In such environment, the effect of lowering borrowing costs in one sector propagates throughout the economy. In our model, therefore, subsidies to a sector have effects that go beyond simply increasing production of such sector. By increasing its production and reducing its prices, the working capital constraints of its downstream sectors are alleviated. Indeed, since their input costs decreases, their demand increases and production gets closer to its frictionless counterpart. This also decreases their prices, which consequently also alleviates their own downstream sectors' working capital constraints.

In general, sectors downstream of the targeted ones experience positive spillovers from the earmarked credit policy program through general equilibrium effects. Therefore, subsidies feature heterogeneous effects on the economy, since sectors face different financial constrains and are not equally relevant in the production network.

There are two forces in the model associated to earmarked credit policy that could justify aggregate TFP movements: labor market distortions and internalizing pecuniary externalities. The first one comes from the need to finance subsidies with distortionary taxes, which consequently causes misallocation of resources at the firm-level and hence a fall in aggregate TFP. The second one comes from the positive spillover that are only (either partially or fully) internalized by sectors via interest rate subsidies. Therefore, a government could lead the economy to higher or lower aggregate TFP levels depending on how much and how well distributed across sectors subsidies are.

Using Brazilian input-output tables and data on sectoral interest rates, we use the model to understand the effect of sectoral productivity and financial frictions on aggregate productivity. Our functional form assumptions for the production sector and the financial frictions introduced in the model allow us to write aggregate productivity that is multiplicatively separable on sectoral productivities and borrowing costs (market interest rates and credit subsidies). We find that the change in earmarked credit policy that occurred from 2008 to 2014 played a modest positive role in increasing GDP per worker and aggregate productivity. The observed policy was effective in raising the economy's efficiency and hence output per worker when compared to a policy that kept sectoral interest-rate subsidies in the levels observed in 2008, and especially in a counterfactual economy with no government subsidies whatsoever. Additionally, the model suggests that credit subsidies should have been even higher than the ones observed in the data. Welfare analysis indicates that implementing such a policy would have increased output per worker and welfare significantly.

2.1.1 Related Literature

The intervention on the credit market by the federal government was mainly to provide firms with cheaper credit than they would in the private credit market. The policy is also partly motivated by the idea that firms should receive subsidized credit due to financial constraints: these firms are either very young or are part of a sector in which the social benefit of their economic activity are bigger than the private one. As Bonomo et al. (2015) find out, however, this credit seems to be directed towards older and larger firms, which would likely be able to finance themselves on the private and free credit market more easily.

Carvalho (2014) also finds that these loans occur more often to companies in regions where local elections are coming, and the incumbent government is less likely to win by the time the loan is approved.

Both papers cited above hint that the use of earmarked credit as a government policy may have generated misallocation of productive resources in Brazil. The magnitude in which such policy affected resource allocation is an open question. Following the methodology in Hsieh and Klenow (2009), Vasconcelos (2017) finds evidence that resource misallocation increased in Brazil from 2005 to 2011. On the other hand, Cavalcanti and Vaz (2017) find that when a firm has access to earmarked credit in Brazil, it increases its investment and productivity if the access is permanent, not temporary.

The literature on the link between financial frictions and misallocation is also extensive. Gilchrist et al. (2013) look at the effect of heterogeneous borrowing cost for firms in the U.S. financial market and find little negative aggregate effects of such heterogeneity. Midrigan and Xu (2014) use a model in which financial frictions affect both technology adoption between firms and dispersion in returns to capital. They find little losses in misallocation due to such frictions. Gopinath et al. (2017) provide evidence of capital misallocation effects of a decline in the real interest rate in Spain in the presence of financial frictions. They find that capital inflows are misallocated toward firms with higher net worth, which aren't necessarily more productive.

Recently, papers such as Acemoglu et al. (2012) emphasize the role of sectoral shocks in driving the aggregate economy. The role of small sectoral distortions in generating substantial macroeconomic effects has also been recently explored in the literature. Baqaee and Farhi (2017) show that the aggregate impact of a sectoral shock can be summarized into two components: a technology shock and its effect on allocative efficiency arising from the reallocation of resources. They show that the latter effect can be quantitatively substantial.

Justification for industrial policies are usually related to the presence of externalities (Atkinson and Stiglitz, 2015). Liu (2017) finds that effects of market imperfections accumulate through *backward demand linkages*, that is, distortions accumulate upstream in a production network. Therefore, a benevolent government should focus on subsidizing sectors which are upstream from highly distorted ones to improve aggregate welfare.

Network externalities can arise from production networks in which firms face financial constraints (Altinoglu, 2018; Bigio and La'O, 2016; Luo, 2018). Under such circumstances, financial constraints in one sector have effects in the rest of the economy via the intermediate demand. A distorted firm will demand less than optimal inputs from its suppliers, which will therefore have depressed sales and will themselves demand less than optimal from their own suppliers.

2.2 The Role of Earmarked Credit in Brazil

In Brazil, a segment of the credit market consists of credit directed towards predetermined sectors or activities, using resources regulated by law or discretionarily allocated by the government. Three main entities provide funds for the lending operations. The BNDES provides credit to private firms, either directly or through commercial banks, for their investment and working capital decisions. Housing financing for households is mostly financed through the public bank Caixa Econômica Federal, while credit for the agricultural sector is mainly provided by the public bank Banco do Brasil.

Since 2008, earmarked lending has steadily increased its share in total credit. Initially implemented as a policy response to the liquidity shortage in the global economy due to the credit crunch of 2008, it later became a government policy tool to bring cheaper credit to firms. In particular, the creation of the Program to Sustain Investment (*Programa de Sustentação do Investimento*, in Portuguese) in mid-2009 made the use of earmarked credit to stimulate private sector's investment a key component for the government's long-term economic development project. As Figure 2.2 shows, the increase in the earmarked credit participation in the total stock of credit in Brazilian's financial system has been steady since 2008 until 2016, when the persistent and ever-increasing budget deficits forced the government to wane the policy. The stock of earmarked credit rose from 11.52 percent of GDP in January 2008 to 26.37 percent in December 2015. By the end of 2015, the amount of earmarked credit in the Brazilian economy had reached about half the stock of credit. As Figure 2.2 also shows, the stock of earmarked credit coming directly from BNDES, not counting its loans made through commercial banks, also substantially increased.

The objective of the earmarked credit is to provide credit at a lower rate than the one available in the private market. For instance, the Long Term Interest Rate (TJLP), the benchmark rate for credit supplied by the BNDES, is set below the monetary policy rate, and frequently reaches negative values in real terms. As Figure 2.3 shows, earmarked rates, from the BNDES or other sources, are on average substantially lower than the ones in the private credit market, and less volatile as well.

This difference is less striking, but still present, when we compare interest rates charged in the earmarked and private credit market for loans with similar characteristics. We investigate this by using a monthly dataset provided by the Central Bank of Brazil that contains data on loans for firms, aggregated by sector, using the 6-digit National Classification of Economic Activities (CNAE) code.[1] Our dataset contains information on the average amount lent, average interest rate, maturity, average value of guarantees, type of loan (earmarked

[1]The Brazilian National Institute of Geography and Statistics (IBGE) has its own industry classification system. It can be mapped into the common ISIC industry classifications.

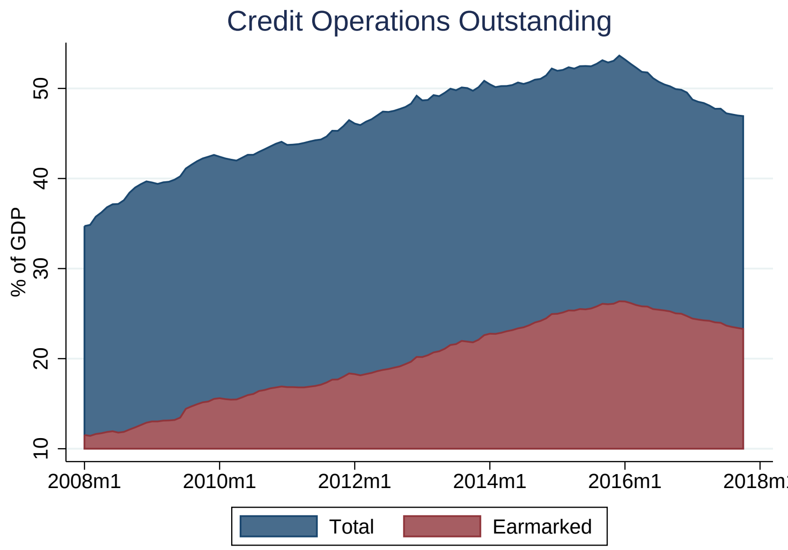


Figure 2.2: Credit Operations Outstanding

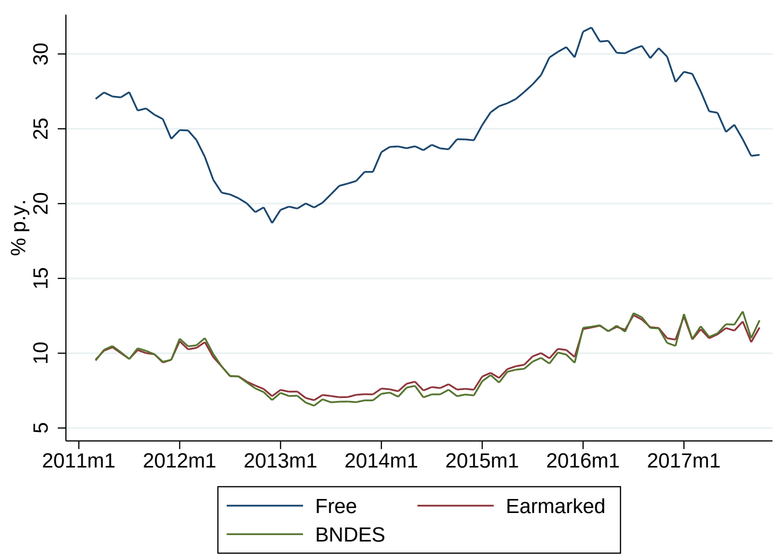


Figure 2.3: Average Interest Rate of New Credit Operations

Non-Financial Corporations

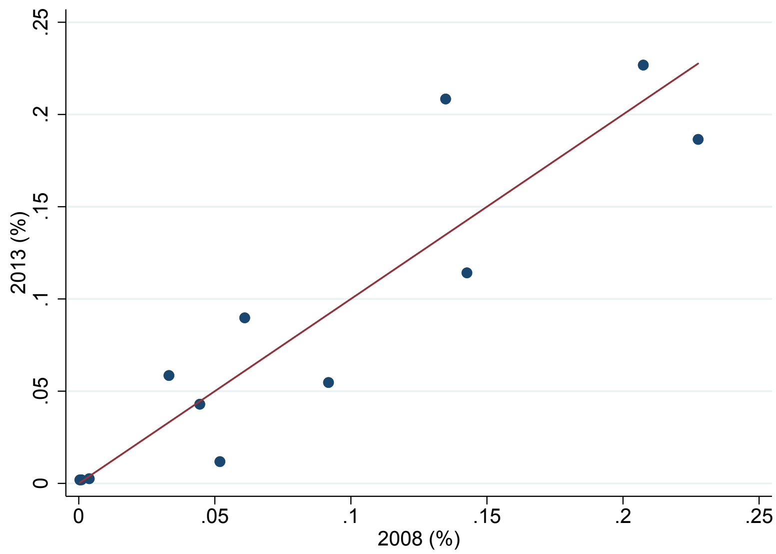


Figure 2.5: Inter-sectoral share of earmarked credit–2008-2013

or nonearmarked), and the category of the loan (working capital, fixed investments, export credits and so on). A regression of interest rates on a dummy for whether the credit is earmarked, controlling for maturity, guarantees, category, year and sector dummies, indicates that interest rates charged on earmarked credit are, on average, ten percentage points lower than the ones in the nonearmarked segment of the credit market.

Using financial data aggregated to the 12-sector division in the Brazilian national accounts,[2] Figures 2.5 and 2.6 show that, even though the policy has increased its role in the Brazilian economy, it did not increase symmetrically across sectors. In particular, Figure 2.5 shows that the share of total earmarked credit in 2008 that was directed to each sector was fairly different than in 2013.

Naturally, these movements could had been simply caused by changes in productivity across sectors over time and, hence, be unrelated to the government's policy. Nevertheless, Figure 2.6 indicates they were, to some extent, a consequence of a new targeting of earmarked credit policy. The earmarked-to-credit-outstanding ratio in each sector has also changed from 2008 to 2013, evidencing that the dynamics in Figure 2.5 should not only be attributed to private sector factors.

Moreover, in order to finance the loans subsidies, the government needs to raise funds. According to Pazarbasioglu-Dutz (2017), demand deposits, special funds, and direct lending from the fiscal sector are the main funding sources of earmarked credit. The funding of earmarked credit in 2015 comes from the following sources:

[2] A description of the sectors used can be found in Section 2.5.

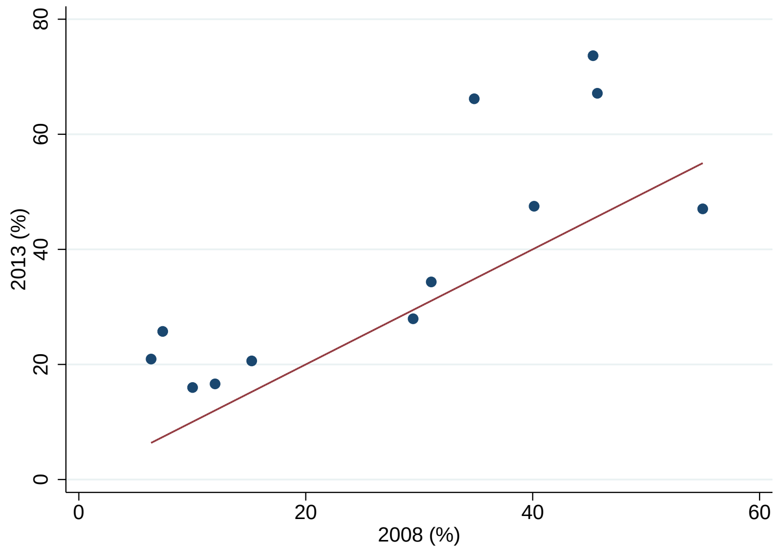


Figure 2.6: Intra-sector share of earmarked credit–2008-2013

1. Savers fund about 40 percent of the earmarked credit. Funding comes from demand and savings deposits as well as tax-exempt financial instruments from specific sectors, such as real estate and agriculture;

2. Employees fund about 12 percent through monthly salary deductions to the Severance Indemnity Fund (FGTS);

3. The fiscal sector funds about 48 percent through direct lending to BNDES and through various special and constitutional funds, such as the Support Fund to the Employee (FAT).

2.3 A Model of Sectoral Linkages with Labor Taxes

We study the impact of earmarked credit in the Brazilian economy using a simple static general equilibrium model with sectoral linkages. The model closely related to the one presented in Bigio and La'O (2016), and especially in Luo (2018), which includes working capital constraints in a model with sectoral linkages and trade credit, where part of the firms' financing costs can be paid after production without borrowing costs. The difference between our model and the one in Luo (2018) is the absence of trade credit, the inclusion of government subsidies, and the introduction of labor taxes, which distorts household's consumption–labor supply decisions.

Even though, as Section 2.2 above discussed, the majority of resources used to finance the earmarked credit policy in Brazil does not come from labor taxes, most of the sources of revenue, if not all, involve some sort of distortion in the private agents' decision-making. For instance, the existence of tax-exempt financial instrument generate distortion in portfolio allocation in the financial markets. The FAT is financed mainly by taxing firms' gross revenues, as well as payroll taxes from non-profit organizations. Increased government borrowing can also be thought of as a potential for distorting consumption-labor supply decisions, either through the use of other distortive taxes or through higher inflation. To keep the model simple, we choose to abstract from most of these details, leave out government debt, and make use of a tax on wages as a way to discipline it.

The economy is composed of N sectors, indexed by $i = 1, ..., N$. Each sector consists of a continuum of competitive firms. Goods are identical across firms within a sector but differentiated across sectors. We index goods by $j = 1, ..., N$, with the understanding that there is a one-to-one mapping between each sector and the good that it produces.

2.3.1 Firms

Firms in sector i produces a specialized product y_i using a Cobb-Douglas production function:

$$y_i = z_i \ell_i^{\alpha_i} \left(\prod_{j=1}^{N} x_{ij}^{\omega_{ij}} \right)^{1-\alpha_i}$$

where x_{ij} denotes the intermediate input used by firm in sector i from sector j; ℓ_i represents the amount of labor input used by the firm i, and z_i is a sector-specific productivity measure. We assume $\sum_{j=1}^{N} \omega_{ij} = 1$ for all i.

There exists a working capital requirement on labor and intermediate inputs. Firms need to borrow in order to pay for their input cost at the beginning of each production period. We assume a small open economy in which firms obtain credit from the rest of the world with a fixed interest rate R, plus an idiosyncratic risk e_i. Policymakers can subsidize the loan on each unit of the credit obtained by the firm in the amount s_i. In this case, the borrowing fee of the firm becomes $R_i = R + e_i - s_i$.

The output of sector i can be used as an intermediate input for other sectors, x_{ji}, as well as for final consumption, which we denote c_i. Due to the financial consraint, the price the firm sells its goods to other firms, p_{ji}, is not the same as the price it sells to households, p_i. Each firm solves the following problem:

$$\max_{\ell_i, \{x_{ij}\}_{j=1}^{N}} p_i c_i + \sum_{j=1}^{N} R_i p_{ji} x_{ji} - \sum_{j=1}^{N} R_i p_{ij} x_{ij} - R_i w \ell_i$$

$$\text{subject to } z_i \ell_i^{\alpha_i} \left(\prod_{j=1}^{N} x_{ij}^{\omega_{ij}} \right)^{1-\alpha_i} \geq y_i = c_i + \sum_{j=1}^{N} x_{ji}$$

2.3.2 Households

The preferences of the representative household are given by:

$$U(c, \ell) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\ell^{1+\varphi}}{1+\varphi}$$

where ℓ denotes labor and c the household's final consumption basket. The final consumption basket of the household is a composite of the differentiated goods in the economy:

$$c = \prod_{j=1}^{N} \left(\frac{c_j}{\nu_j} \right)^{\nu_j}$$

where the parameter $\nu_j \in [0, 1]$ is the household's expenditure share on good j. If the household does not consume good j, then $\nu_j = 0$. Without loss of generality we set $\sum_{j=1}^{N} \nu_j = 1$. The budget constraint of the household is given by:

$$Pc \leq (1-\tau) w\ell + \sum_{i=1}^{N} \pi_i$$

where the left hand side is total expenditure and the right hand side includes the after-tax wage income of the household and dividends from owning all of the firms. P is the price level, given by $P \equiv \Pi_i p_i^{\nu_i}$, which comes from the expenditure minimization problem.

2.3.3 Government

The government taxes labor in order to finance their sectoral loan subsidy policy. We assume it can not borrow in order to finance its expenditures. Therefore, the government chooses a tax rate in order to satisfy its budget constraint.

$$\sum_{i=1}^{N} s_i \left[\sum_{j=1}^{N} (p_{ij} x_{ij} - p_{ji} x_{ji}) + w\ell_i \right] = \tau w \ell$$

2.3.4 Market clearing

The output of any given sector may be either consumed by the household or used by other sectors as an input to production. Commodity market clearing for each good i is thus given by:

$$y_i = c_i + \sum_{j=1}^{N} x_{ji}$$

Similarly, labor market clearing satisfies $\sum_{i=1}^{N} \ell_i = \ell$.

2.3.5 Equilibrium

Given the environment described above, the equilibrium in this model is defined as follows.

Definition 2 *Given the sector-specific interest rate and subsidy,* $(R_i, s_i)_{i=1}^N$*, a competitive equilibrium consists of a vector of commodity prices,* $(p_i)_{i=1}^N$*, a consumption bundle,* $(c_i)_{i=1}^N$*, and sectoral output, intermediate goods, and labor allocations* $\left(y_i, (x_{ij})_{j=1}^N, \ell_i\right)_{i=1}^N$*, such that:*

1. *the household and firms are at their respective optima;*

2. *prices and wages clear the commodity and labor markets.*

2.4 Financial Frictions and Aggregate Distortions

Given the amount of labor supplied by the household ℓ, Proposition 1 below characterizes the equilibrium aggregate value added, tax rate, wages and sectoral prices.

Proposition 1 *Let* $\circ$ *and* $\oslash$ *denote the Hadamard (entrywise) product and division, respectively. Define the vectors* $\mathbf{s} \equiv [s_1 \dots s_N]'$, $\mathbf{R} \equiv [R_1 \dots R_N]'$, $\mathbf{z} \equiv [z_1 \dots z_N]'$, $\boldsymbol{\alpha} \equiv [\alpha_1 \dots \alpha_N]'$*, and* $\boldsymbol{\nu} \equiv [\nu_1 \dots \nu_N]'$*. Let* $\Theta_i \equiv \alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i} \prod_{j=1}^N \omega_{ij}^{(1-\alpha_i)\omega_{ij}}$ *and* Ω *and* Λ *be matrices with entries* $\Omega_{ij} = (1-\alpha_i)\omega_{ij}$ *and* $\Lambda_{ij} = R_i/R_j$*, respectively. In addition, define* $\beta \equiv (I - \Omega')^{-1}\boldsymbol{\nu}$*.*

The aggregate production function of this economy is linear in labor, and given by

$$GDP(\mathbf{z}, \mathbf{R}, \mathbf{s}) = A(\mathbf{z})\mu(\mathbf{R}, \mathbf{s})\ell, \tag{2.1}$$

where the functions $A(\mathbf{z})$ *and* $\mu(\mathbf{R}, \mathbf{s})$ *are given by*

$$A(\mathbf{z}) \equiv \prod_i \Theta_i^{\beta_i} z_i^{\beta_i}$$

$$\mu(\mathbf{R}, \mathbf{s}) \equiv \frac{\prod_i R_i^{-\alpha_i\beta_i} \prod_{j=1}^N \left(\frac{R_i}{R_j}\right)^{-\omega_{ij}(1-\alpha_i)\beta_i}}{1 + \left((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right)\left(I - \Omega' \circ \Lambda\right)^{-1}\boldsymbol{\nu}}.$$

The endogenous tax rate that is consistent with the government policy $\mathbf{s}$ *is given by*

$$\tau = \frac{\left((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right)\left(I - \Omega' \circ \Lambda\right)^{-1}\boldsymbol{\nu}}{1 + \left((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right)\left(I - \Omega' \circ \Lambda\right)^{-1}\boldsymbol{\nu}}. \tag{2.2}$$

The log of the wage rate is given by

$$\log w = \beta' \left[\log \Theta + \log \mathbf{z} - (I - \Omega) \log \mathbf{R}\right]. \tag{2.3}$$

Aggregate price level is normalized to one ($P = 1$). The log of the sectoral price vector $\mathbf{p}$ *is given by*

$$\log \mathbf{p} = \left[I - (I - \Omega)^{-1} \boldsymbol{\alpha} \beta' (I - \Omega)\right] \log \mathbf{R} - (I - \Omega)^{-1} (I - \boldsymbol{\alpha} \beta')(\log \Theta + \log \mathbf{z}). \tag{2.4}$$

2.4.1 The Efficiency Wedge

Proposition 1 shows that the production function aggregates into one that is linear on labor. This aggregate output function is similar from the one in Luo (2018) and in Bigio and La'O (2016). The difference between ours and the latter is our assumption of constant returns to scale. The aggregate production function contains an aggregate TFP that can be decomposed into two terms–one that is a function of sectoral productivity, $A(\mathbf{z})$, and one that depends on the distribution of interest rate spreads and government subsidies, $\mu(\mathbf{R}, \mathbf{s})$. The former is the common aggregation of sectoral productivity, while the latter is the efficiency wedge, i.e., the effect of sectoral distortions on aggregate productivity.

The Impact of a Sectoral Productivity Shock on Aggregate TFP

It's easier to see how aggregate TFP moves due to a sectoral productivity shock by taking logs on $A(\mathbf{z})$ and using the definition of β:

$$\log A(\mathbf{z}) = (I - \Omega')^{-1} \boldsymbol{\nu} (\log \Theta + \log \mathbf{z}).$$

The term $(I - \Omega')^{-1}$ is called the Leontief Inverse matrix. It encapsulates the infinite impact a sectoral productivity shock has in the economic network: a positive productivity shock in sector i simultaneously decreases the sector's price and increases its output. This in turns induces sectors that demand i's intermediate good to purchase more from this sector, increasing their production as well as decreasing their prices. These effects propagate through the economic network, and the magnitude of this effects depends on the network structure in Ω.

The Impact of Government Subsidies on Aggregate TFP

To build intuition understand the impact of government subsidies on aggregate productivity, we split the discussion in three parts.

The impact of borrowing costs on TFP. Assume no subsidies and that borrowing costs are equal across sectors and $R > 1$. It can be shown that $\boldsymbol{\alpha}'\beta = 1$, so therefore the efficiency wedge can be expressed as follows:

$$\log \mu(\mathbf{R}, \mathbf{s}) = -\log R.$$

In this case, the effect of borrowing costs on aggregate productivity is represented by a downward shift in the aggregate production function.

The impact of dispersions in borrowing costs on TFP. Assume no subsidies, but now borrowing costs are allowed to vary across sectors. The efficiency wedge can be expressed as follows:

$$\begin{aligned}
\log \mu(\mathbf{R}, \mathbf{s}) &= -\sum_i \beta_i \log R_i + \sum_i \left[\sum_j (1-\alpha_j)\omega_{ji}\beta_j \right] \log R_i \\
&= -\sum_i \left[\beta_i + \sum_j (1-\alpha_j)\omega_{ji}\beta_j \right] \log R_i \\
&= -(\beta + \Omega'\beta)' \log \mathbf{R} \\
&= -\beta'(I+\Omega) \log \mathbf{R}.
\end{aligned}$$

In this setting, the network structure of the economy plays a role in shifting the aggregate TFP downwards–a higher borrowing cost in sector i distorts the first-order condition for intermediate goods in all sectors that demand i's output. This creates resource misallocation, which is amplified through the network structure of the economy.

The impact of borrowing costs subsidies on TFP. When we add government subsidies to borrowing costs, the efficiency wedge can be expressed as follows:

$$\begin{aligned}
\log \mu(\mathbf{R}, \mathbf{s}) = &- \beta'(I+\Omega) \log \mathbf{R} \\
&- \log \left(1 + ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))') \left(I - \Omega' \circ [(\mathbf{R}\mathbf{1}') \circ (\mathbf{1}\mathbf{R}')] \right)^{-1} \boldsymbol{\nu} \right),
\end{aligned}$$

where $\mathbf{1}$ is a vector of ones and the fact that Λ can be written as $(\mathbf{R}\mathbf{1}') \circ (\mathbf{1}\mathbf{R}')$.

Note that, in our definition, $R_i = R + e_i - s_i$. Thus, it's hard to make comparative statics using the expression above. Nevertheless, we can see from above that if the term $((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))') \left(I - \Omega' \circ [(\mathbf{R}\mathbf{1}') \circ (\mathbf{1}\mathbf{R}')] \right)^{-1} \boldsymbol{\nu}$ is positive, government subsidies have deleterious effects on aggregate productivity.

To think about how government subsidies affect the economy's aggregate productivity, start by assuming initially that there are no subsidies to borrowing costs, so that $s_i = 0$ for all $i \in \{1, \ldots, N\}$ and thus $\tau = 0$. Denote the vector of the log of the sectors' borrowing costs as $\log \mathbf{R}^0$. A necessary and sufficient condition for a given vector of sectoral subsidies $\mathbf{s}$ to improve aggregate TFP is that $\log \mu(\mathbf{R}, \mathbf{s}) - \log \mu(\mathbf{R}^0, \mathbf{0}) > 0$, or

$$
\begin{aligned}
&- \beta'(I+\Omega)(\log \mathbf{R} - \log \mathbf{R}^0) \\
&- \log \left(1 + ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))')\left(I - \Omega' \circ [(\mathbf{R}\mathbf{1}') \circ (\mathbf{1}\mathbf{R}')]\right)^{-1} \boldsymbol{\nu}\right) > 0.
\end{aligned}
$$

2.4.2 The Absence of the Labor Wedge

Now, we close the model using the household's optimality condition to derive the labor wedge. The optimality condition for the household is given by

$$
\frac{\ell^{\varphi}}{c^{-\sigma}} = (1-\tau)w.
$$

The labor wedge, $1 - \tau_\ell$, is implicitly defined by the relation

$$
\frac{\ell^{\varphi}}{c^{-\sigma}} = (1 - \tau_\ell(\mathbf{s}, \mathbf{R}))\frac{c}{\ell}.
$$

Combining both, we get

$$
(1 - \tau_\ell(\mathbf{s}, \mathbf{R}))A(\mathbf{z})\mu(\mathbf{R}, \mathbf{s}) = (1-\tau)w.
$$

Plugging in the functional forms for the tax rate, wages, sectoral productivity and the efficiency wedge, we get the following result.

Proposition 2 *In equilibrium, the economy's aggregate labor wedge is always equal one. That is,* $\tau_\ell(\mathbf{s}, \mathbf{R}) = 0$.

The reason why our model does not contain the labor wedge rests solely on the fact that we are not allowing the interest-rate spread e_i to return to the household as a lump-sum transfer. Here, the revenue generated from those spreads are simply thrown into the ocean. In this setting, because the sectoral production function is constant returns to scale and all the distortions are incorporated in the after-tax wage rate, the household has no other source of income to finance consumption. In Appendix B, we derive the labor wedge for the case in which the revenue generated from the interest-rate spreads are rebated back to the consumer. In such setting, both the efficiency wedge and the tax rate change compared to our baseline model.

2.4.3 Equilibrium path

Starting from the labor wedge definition, we get

$$
\ell^{1+\varphi} = (1 - \tau_\ell(\mathbf{s}, \mathbf{R}))c^{1-\sigma}.
$$

Given that $c(\mathbf{z}, \mathbf{R}, \mathbf{s}) = A(\mathbf{z})\mu(\mathbf{R}, \mathbf{s})\ell$, we take logs and arrive at the following lemma:

Lemma 3 *Given sectoral productivity, efficiency and labor wedges, equilibrium aggregate consumption and labor supply are given by*

$$\log \ell(\mathbf{R}, \mathbf{s}) = \frac{1-\sigma}{\sigma+\varphi}(\log A(\mathbf{z}) + \log \mu(\mathbf{R}, \mathbf{s})) + \frac{1}{\sigma+\varphi}\log(1-\tau_\ell(\mathbf{s}, \mathbf{R})) \tag{2.5}$$

$$\log c(\mathbf{R}, \mathbf{s}) = \frac{1+\varphi}{\sigma+\varphi}(\log A(\mathbf{z}) + \log \mu(\mathbf{R}, \mathbf{s})) + \frac{1}{\sigma+\varphi}\log(1-\tau_\ell(\mathbf{s}, \mathbf{R})). \tag{2.6}$$

2.4.4 Intuition Behind the Model

In the model, the government needs to raise funds in order to subsidize firms via distortionary labor taxes. Whenever a firm's borrowing costs are subsidized, it increases its input demand and, consequently, its output. By supplying more goods, this sector's price is reduced. Therefore, the subsidy works as if it was alleviating the borrowing constraint of firms in all the downstream sectors from the original firm: all its customers purchases their intermediate goods at lower prices. Consequently, all downstream sectors produce more and sell their output at lower prices, allowing their own downstream sectors to purchase more intermediate goods, produce more, and sell their output at lower prices. This effect goes on forever, which implies that total output in this economy increases due to lower interest rates.

Therefore, this multi-sector economy with borrowing constraints features externalities that are only internalized with government subsidizes. Subsidy will only be beneficial to the economy when the spillover of the targeted sector towards its downstream sectors compensates the labor market distortion.

2.5 Calibration

We use annual data on National Accounts provided by the Brazilian Institute of Geography and Statistics (IBGE), in particular their Use tables, disaggregated at 12 industries for the years 2000-2015. The industries are: Agriculture, Extraction Industries, Manufacturing, Utilities, Construction, Retail, Transportation and Storage, ICT, Financial Services and Insurance, Real Estate, Other Services and Public Administration. Because we assume sectors borrow from the international financial markets, we exclude Financial Services and Insurance from our quantitative exercise. For each year, this table identifies sector i's uses (expenditures) of goods produced by sector j, the empirical analogue of $p_{ij}x_{ij}$. Final use of sector i net of gross fixed capital formation and inventories is the empirical analogue of $p_i c_i$. For the empirical analogue of $w\ell_i$, we add to sector i's labor compensation half of the sector's gross mixed income. This is motivated by the fact that this item in the national accounts includes components of both labor and capital income. We take the agnostic stance of assigning assigning half of such income to labor.

2.5.1 Input shares

Due to the Cobb-Douglas production function of intermediate goods and the Cobb-Douglas final goods aggregator, this data is sufficient for computing the labor shares, α_i, the intermediate inputs shares, ω_{ij}, and the consumption shares, ν_i. Therefore, we compute these parameters for each year in our sample using the following model's first order conditions:

$$\alpha_i = \frac{w\ell_i}{w\ell_i + \sum_{k=1}^{N}(p_{ik}x_{ik})}$$

$$\omega_{ij} = \frac{p_{ij}x_{ij}}{\sum_{k=1}^{N}(p_{ik}x_{ik})}$$

$$\nu_i = \frac{p_i c_i}{\sum_{k=1}^{N}(p_k c_k)}.$$

We also retrieve a sector-specific price index from the Make tables, since they are available in current prices as well as previous-year prices. We observe, therefore, the empirical analogue of (p_{ij}), which allows us to recover each sectors' intermediate input demand (x_{ij}) and output (y_i). IBGE also reports the number of employers in each sector, which is the empirical analogue of ℓ_i. This suffices to compute sector-specific productivities:

$$z_i = \frac{y_i}{\ell_i^{\alpha_i}\left(\prod_{j=1}^{N} x_{ij}^{\omega_{ij}}\right)^{1-\alpha_i}}.$$

We chose to let parameters vary by year instead of taking the yearly average due to the nature of the Cobb-Douglas function. The value of intermediate demand from some sectors is zero in some years. If the parameters ω_{ij} are constant, in the years where x_{ij} takes the value of zero, the entire expression $\ell_i^{\alpha_i}\left(\prod_{j=1}^{N} x_{ij}^{\omega_{ij}}\right)^{1-\alpha_i}$ would take the value of zero as well. Nevertheless, they are fairly stable. Figure 2.7 depicts the sectoral labor share across the years.

2.5.2 Sectoral subsidies

The Central Bank of Brazil manages a loan-level database called Credit Information System (SIC) that consists of all loans above R$5,000 (during the 2008-2015 period, its dollar-equivalent nominal value was always in the $1,250–3,000 range). It provides information on loan amount, maturity, collateral value, interest rate, industry, and category (project financing, vehicle financing, bill discounting, etc). Our data set is monthly and spans from the years 2004 to 2018. Markedly, it also discriminates earmarked from non-earmarked loans. Therefore, our sample offers a good representation of the entire Brazilian economy, suitable for analyzing earmarked credit policy.

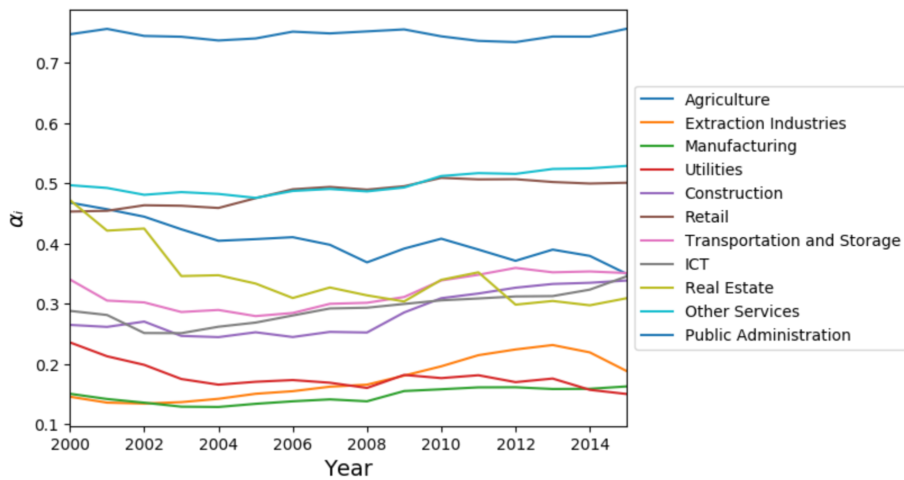


Figure 2.7: Labor share (α_i) by sector

We compute the sum of world interest rate plus idiosyncratic risk, $R + e_i$, as the sector i's average real interest rate of non-earmarked loans from this database. Analogously, we use the sector i's average real interest rate of earmarked loans as the sum of world interest rate plus idiosyncratic risk minus subsidy rate, $R + e_i - \varsigma_i$. Sector i's subsidy rate, ς_i, is then calculated as the difference between these two objects. We also extract, for each sector i, the share of total loans that are earmarked, ϕ_i, directly from the data. We use these two objects to compute the subsidy by each unit of credit to sector i as $s_i = \phi_i \varsigma_i$.

In our quantitative exercise, we choose to focus on the transition between the years 2008 and 2013. We chose 2008 since it's the last year before the government started using earmarked lending as a countercyclical measure to deal with the 2008 financial crisis. In 2014, the presidential election took place, and the economy started to experience a slowdown. The latter may have been affected by other factors outside our model, which is why we decided to use 2013 instead of 2014. Table 2.1 presents summary statistics for the interest rate spread and share of earmarked credit in the eleven sectors. The table shows that, on average, the reduction in interest rate per sector from the earmarked lending decreases from 2008 to 2013, while the share of earmarked lending in each sector significantly increased. A reason why the interest rate spread is smaller in 2013 than in 2008 on average is potentially explained by the decrease in the average lending rate in Brazil due to monetary easing in Brazil and in the world: the rate was on average 47.25% in 2008, and 27.39% in 2013.

2.6 Quantitative Exercises

This section presents the main quantitative findings of the chapter. We first investigate the role of the dynamics of sectoral productivity to aggregate productivity. Next, we use

Table 2.1: Productivity and Credit Data by Sector

	Interest rate spread (p.p.)		Share of earmarked credit (%)	
Sector	2008	2013	2008	2013
Agriculture	5.27	4.68	41.64	48.27
Extraction Industries	3.26	5.22	57.25	47.42
Manufacturing	4.48	3.76	15.22	20.56
Utilities	-1.03	2.02	46.09	72.95
Construction	11.03	10.59	30.36	34.08
Retail	4.78	4.95	9.24	15.87
Transportation & Storage	11.7	13.09	45.56	67.39
ICT	13.85	-6.14	3.96	21.14
Real Estate	6.22	9.05	11.57	16.58
Other Services	13.77	4.25	27.57	28.39
Public Administration	16.08	13.03	4.67	25.77

Note: The interest rate spread, ς_i, represents the difference, in percentage points, between the interest rates charged for nonearmarked and earmarked loans in sector i. The share of earmarked credit, ϕ_i, represents the share or earmarked loans in sector i in the given year.

the model to assess the optimal vector of sector subsidies given exogenous sector-specific interest rates observed in the data, as well as considering the counterfactual policy of not intervening in the credit markets through subsidy, consequently not taxing labor income.

2.6.1 Decomposing aggregate productivity

Using equation (2.1), we can infer the contribution of the sectoral productivity component of TFP to aggregate productivity. The left panel of Figure 2.8 plots the function $A(\mathbf{z}_t)$ for years 2000 to 2015. The values were normalized so that $A(\mathbf{z}_{2008}) = 1$. The figure shows a productivity slowdown between the years 2000 and 2003, followed by a recovery from 2004 to 2008. After the Great Recession in the United States in 2008, there was a sharp drop of more than six percent in the sectoral productivity component of TFP in 2009, followed by a quick recovery in 2010 and 2011. By the latter year, the sectoral productivity component of TFP was about four percent higher than in 2008. From 2012 onwards, this component of aggregate productivity starts to fall considerably. In 2015, it was about eight percent lower than in 2008.

We can use equation (2.1) to decompose the dynamics of the efficiency wedge as well. The right panel of Figure 2.8 plots the function $\mu(\mathbf{R}_t, \mathbf{s}_t)$ for years 2004 to 2015. we observe that it contributed positively to aggregate TFP growth between 2004 and 2008. In 2009, it contributed negatively, just as the sectoral productivity component. From 2010 to 2012, it again contributed to productivity growth, contributing negatively afterwards.

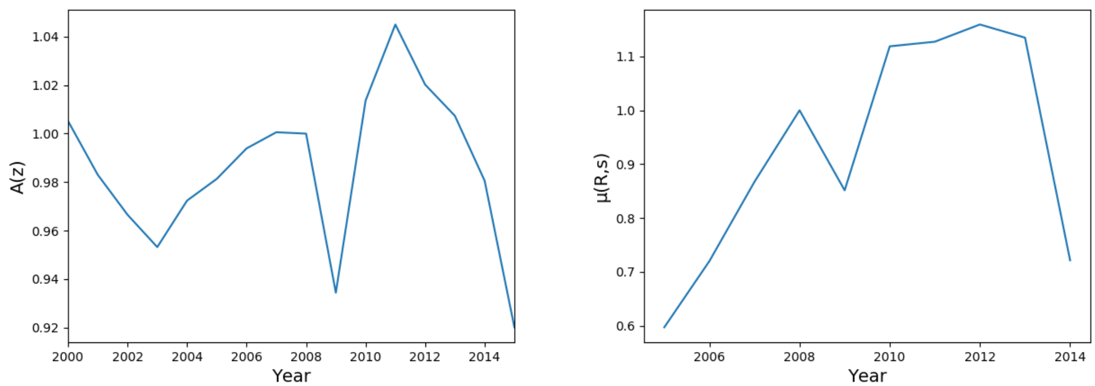
If we apply a comparative statics exercise, analyzing the contribution of each component to GDP per worker growth from 2008 to 2013, the year before the presidential elections, the model indicates that aggregate TFP grew 0.73% due to sectoral productivity growth, and 13.48% due to the efficiency wedge. This implies that factors affecting the behavior of each sector's productivity were much more important to explain why Brazil did not grow as fast after recovering from the spillovers of the Great Recession in the United States.

From 2013 onwards, both the sectoral productivity component and the efficiency wedge were responsible for a sharp drop in aggregate productivity. In particular, the efficiency wedge dropped as much as forty percent from 2013 to 2014. This can be partly explained by the increase in world interest rates.[3] In the next section, we will investigate whether the government policy was responsible for such a sharp drop in this component of TFP.

2.6.2 Optimal interest rate subsidy policy

In this section, we use the optimal consumption and labor derived from the household optimality conditions, (2.5) and (2.6), to choose the vector of government subsidies that

[3] For instance, five-year U.S. Treasury bonds increased half a percentage point.



(a) Sectoral productivity (b) Efficiency wedge

Figure 2.8: Contributions to aggregate productivity (2008=1)

maximizes household utility. For this exercise, we choose $\sigma = 2$ and $\varphi = 0.5$, common values in the business cycle literature.

Figure 2.9 compares the observed subsidy policy to the optimal one for the years 2008, 2010 and 2013 and is the most important finding in this chapter. Each dot represents the observed and optimal subsidy for a single sector in one given year We find that, through the lens of our model, the optimal subsidy policy should generally be much higher than the one observed in the data. In general, lower subsidies were required for 2013 when compared to 2008, even though they should have been substantially higher than the actual policy in both years. This is in line with the discussion in Section 5.2. The decrease in the average lending rate in Brazil due to monetary easing in Brazil and in the world required a less aggressive subsidy policy from the government.

Next, we analyze overall welfare gains of implementing the optimal policy from 2008 to 2013. Figure 2.10 plots the results. We find that applying the optimal policy improves welfare substantially, especially between 2008 and 2009. In particular, would have been about twenty five percent higher in 2009 if the optimal policy was implemented compared to the actual policy.

Figures 2.11 and 2.12 plot the behavior of the efficiency wedge and GDP per worker. Both measures would be at least twenty percent higher than the actual observed value in 2008 in every year of the sample. The optimal policy would have offset the negative contribution of the sectoral productivity and the efficiency wedge components of TFP more substantially.

We also perform two counterfactuals: fixing the government policy to the one observed in 2008, as well as removing all sectoral subsidies. Figure 2.10 presents these results. In terms of welfare gains, both policies would have been welfare-reducing, especially the one where the government removes all sectoral subsidies. Focusing on freezing the policy

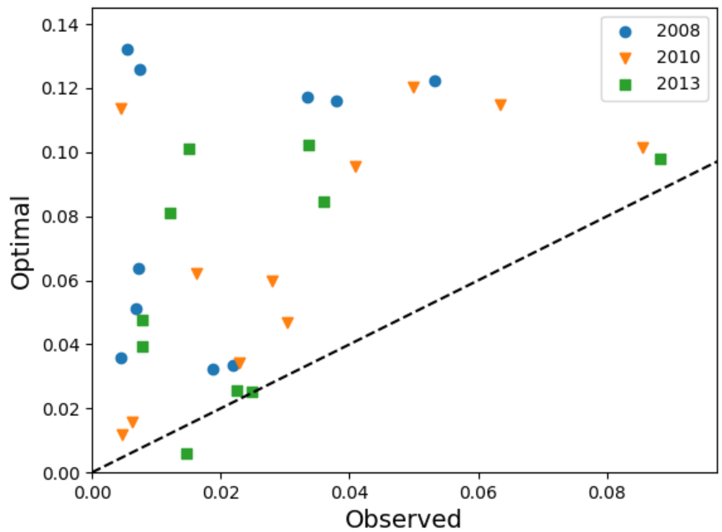


Figure 2.9: Optimal vs. observed policies

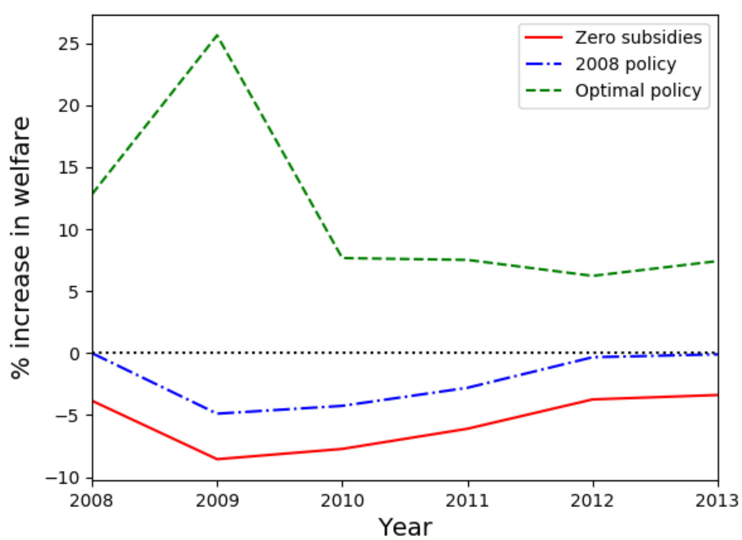


Figure 2.10: Welfare relative to actual policy

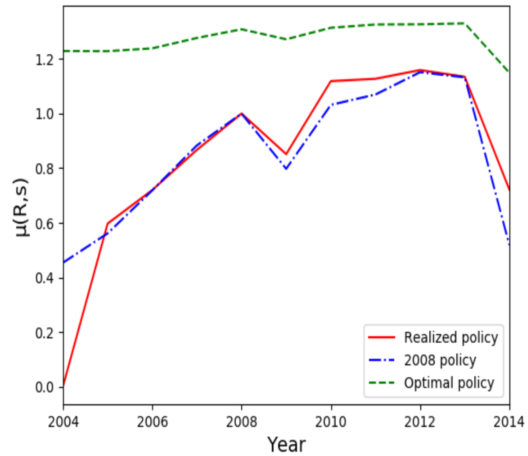


Figure 2.11: Efficiency wedge

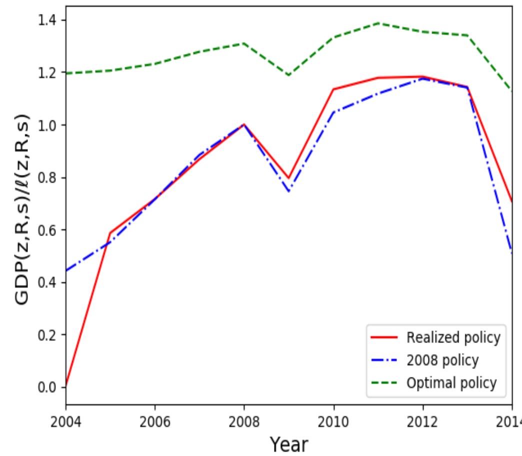


Figure 2.12: GDP per worker

to 2008 levels, Figures 2.11 and 2.12 indicate that the behavior of the efficiency wedge and output per worker would have been roughly similar to the data from 2004 to 2015. The changes in credit policy from 2008 onwards were not as relevant as the behavior of sectoral productivities to explain the behavior of aggregate TFP. Nevertheless, the policy implemented by the government contributed to offset the negative effects of interest rate shocks on the efficiency wedge.

2.7 Conclusion and Directions for Further Research

Under the lens of our model, the change in earmarked credit policy that occurred from 2008 to 2014 played a positive, albeit modest, role in the economy in terms of GDP per worker. The observed policy was effective in raising the economy's efficiency and hence output per worker when compared to a policy that kept sectoral interest-rate subsidies in the levels observed in 2008, and especially in a counterfactual economy with no government subsidies whatsoever. More importantly, the model suggests that the government should have provided sectors with even more subsidies than the ones observed in the data. Implementing such a policy would have increased output per worker and welfare significantly.

A potential model is one in which the government can finance its credit policy not only by taxing labor, but also by borrowing. In this setting, expanding earmarked loans may increase the government's default risk. If the idiosyncratic borrowing cost faced by each sector is correlated with the government's, sharp increases in earmarked credit policy force the government to increase both labor taxes and debt, which imposes not only labor distortions but also higher borrowing costs for non-earmarked loans.

Even though our current model is not appropriate for addressing such channel, it does illustrate one of our next steps. To our knowledge, sovereign default risk was never used in a general equilibrium model for modeling the costs of industrial policies. Or, similarly, earmarked loans in an economy with sectoral linkages were never used to explicitly model rolling over debt costs.

Another challenge to our theoretical framework and therefore our conclusions in this chapter is the absence of capital as a factor of production. Because we do not have sectoral data on fixed assets from the national accounts, it's not feasible to introduce capital accumulation to the current setting. Since a substantial share of the earmarked loans were directed towards long-term investments, omitting the investment channel in our analysis may have significantly affected our conclusions. One potential solution is to use the Annual Survey of Industry (PIA). Conducted by the IBGE, it is a yearly census of the Brazilian manufacturing sector. This data set covers all firms in the manufacturing and extractive industries, including firm-level data on labor compensation, capital expenditures, fixed assets and purchases of intermediate goods. Even though it would restrict the analysis to a subset of sectors of the Brazilian economy, the potential gains of providing more detailed analysis of the role of earmarked credit in the firms' investment decisions could therefore shed some light in the role of the government policy in capital (mis)allocation across these sectors.

Additionally, our current analysis is silent about the effect of this policy on industry concentration. In Section 2.6.1, we provided evidence that the decline in sectoral productivity was a relevant driver of the slowdown in aggregate TFP. It is possible that assuming a representative firm in each sector may be hiding interesting intra-sector firm dynamics. As shown by Bonomo et al. (2015), inside each industrial sector, earmarked loans were directed towards older and bigger firms. The government policy may be leading to deleterious market concentration by allocating cheaper capital to bigger, unproductive firms, while forcing younger and productive firms to exit the market, thus lowering average productivity by sector. Again, using the PIA could answer this question for the manufacturing and extractive industries.

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Appendix A

Appendix to Chapter 1

A.1 Tract-Level Housing Productivity and Zoning Reform

Suppose there's a developer that wants to build in a parcel with land size L_i in location j. They combine L_i with materials M_i to create total square footage in the parcel, with encompasses both floorspace by unit and number of units in the parcel. Developer chooses both u_i, the number of units in the parcel, and a_i, floorspace by unit. The parcel-level productivity is given by g_i. Their maximization problem is:

$$\max_{u,M} r_j u g_i L_i^{(1-\phi)} M^{\phi} - \iota M - p_j \left(\frac{u}{L_i} \right).$$

Given u, the supply of floorspace per unit is given by $g_i^{\frac{1}{1-\phi}} u^{\frac{\phi}{1-\phi}} \left(\frac{\phi r_j}{\iota} \right)^{\frac{\phi}{1-\phi}} L_i$. After maximizing with the respect to M, the developer then picks $u \in 0, 1, \ldots, \overline{u}$ that maximizes profits. Denote this u as $u^*(\overline{u})$. The upper bound, $\overline{u}$, is dictated by local zoning laws. Similarly, suppose there is a representative developer at the tract level, with productivity G_j and land L_j. The supply function of this developer is given by $G^{\frac{1}{1-\phi}} \left(\frac{\phi r_j}{\iota} \right)^{\frac{\phi}{1-\phi}} L_j$. The sum of total housing supplied by land, in both cases, is given by

$$\sum_{i \in j} h_i = \left(\frac{\phi r_j}{\iota} \right)^{\frac{\phi}{1-\phi}} \sum_{i \in j} g_i^{\frac{1}{1-\phi}} u_i^*(\overline{u})^{\frac{\phi}{1-\phi}} L_i$$

$$H_j = \left(\frac{\phi r_j}{\iota} \right)^{\frac{\phi}{1-\phi}} G_j^{\frac{1}{1-\phi}} L_j.$$

If we want to use the aggregate housing production function to represent the sum of housing supply by lot, internal consistency requires that

$$G_j = \left[\sum_{i \in j} \frac{L_i}{L_j} g_i^{\frac{1}{1-\phi}} u_i^*(\overline{u})^{\frac{\phi}{1-\phi}} \right]^{1-\phi}.$$

That is, tract-level productivity is the land-weighed sum of lot-level productivity and optimal unit choice. For the counterfactual exercise presented in the main body of the text to make sense conceptually, it is sufficient that the choice of $u^*(\overline{u}_{pre}) < u^*(\overline{u}_{post})$, where $\overline{u}_{pre}$ is the upper bound on development before the zoning reform and $\overline{u}_{post}$ is the upper bound after the zoning reform. In such case, an increase in number of units per lot is equivalent to a productivity increase in the tract level.

A.2 Parameters

Parameter	Value	Source
α	0.76	Davis and Ortalo-Magne (2011)
β	0.8	Ahlfeldt et al. (2015)
η	0.06	Ciccone and Hall (1996)
ϕ	0.3	Severen (2018)
ψ	6.7	Severen (2018)
θ	6.99	Regression
ι	0.06	Owens III et al. (2020)

Table A.1: Parameter Values

A.3 Further Model Derivations

A.3.1 Worker's Location Choice

Before exploring the problem, it is convenient to define the following probability:

$$G_{ij}(v) = \Pr(\hat{V}_{ij} \leq v).$$

Let $\psi_{ij} \equiv a_{ij} V_{ij}^{\theta}$. Using the definition of counterfactual indirect utility above and the functional form of the Fréchet distribution, we have:

$$G_{ij}(v) = \Pr\left(\epsilon_{ij} \leq \frac{v}{V_{ij}}\right) = F_{ij}\left(\frac{v}{V_{ij}}\right) = \exp\left(-\psi_{ij} v^{-\theta}\right).$$

Similarly, by independence of the draws,

$$
\begin{aligned}
\Pr\left(\max_{i,j\in\Omega}\{\hat{V}_{ij}\} \leq v\right) &= \Pr\left(\cap_{i,j\in\Omega}\left(\hat{V}_{ij} \leq v\right)\right) \\
&= \prod_{i,j\in\Omega} \Pr\left(\hat{V}_{ij} \leq v\right) \\
&= \prod_{i,j\in\Omega} G_{ij}(v) \\
&= \prod_{i,j\in\Omega} \exp\left(-\psi_{ij} v^{-\theta}\right) \\
&= \exp\left(-v^{-\theta} \sum_{i,j\in\Omega} \psi_{ij}\right).
\end{aligned}
$$

From the law of large numbers, the fraction of workers living in location j and working in neighborhood i, π_{ij}, can be represented by:

$$
\begin{aligned}
\pi_{ij} = \Pr\left(\hat{V}_{ij} \geq \max_{i',j'}\{\hat{V}_{i'j'}\}\right) &= \int_0^\infty \prod_{i',j'\in\Omega} G_{i'j'}(v) dG_{ij}(v) \\
&= \int_0^\infty \exp\left(-v^{-\theta} \sum_{i',j'\in\Omega} \psi_{i'j'}\right)\left(\psi_{ij}\theta v^{-\theta-1}\right) dv \\
&= \psi_{ij} \int_0^\infty \theta v^{-\theta-1} \exp\left(-v^{-\theta} \sum_{i',j'\in\Omega} \psi_{i'j'}\right) dv \\
&= \psi_{ij} \left[\frac{\exp\left(-v^{-\theta}\sum_{i',j'\in\Omega}\psi_{i'j'}\right)}{\sum_{i',j'\in\Omega}\psi_{i'j'}}\right]_0^\infty = \frac{\psi_{ij}}{\sum_{i',j'\in\Omega}\psi_{i'j'}} \\
&= \lambda a_{ij}\left(\kappa_{ij} r_j^{(1-\alpha)}\right)^{-\theta}(w_i s_j)^\theta.
\end{aligned}
$$

where $\lambda \equiv \left[\sum_{i',j'\in\Omega}\psi_{i'j'}\right]^{-1}$.

The equation above is a gravity equation for commuting, describing overall patterns of workers' workplace and location choices. It shows that the fraction of the population living in j and working in i is increasing in the location taste shock a_{ij}, wages paid in i, and amenities in j. Similarly, the share of workers is decreasing in costly it is to commute between the pair ij, how high are residential taxes in j, and rent (r_j). Sensitivity to these variables depend on shape parameter θ of location taste

Summing across residential locations, we get the share of workers in location i:

$$
\pi_i = \sum_{j'\in\Omega} \pi_{ij'} = \lambda \sum_{j'\in\Omega} a_{ij'} V_{ij'}^\theta.
$$

The share of workers living in location j is given by:

$$\pi_j = \sum_{i' \in \Omega} \pi_{i'j} = \lambda \sum_{i' \in \Omega} a_{i'j} V_{i'j}^{\theta}.$$

Equivalently, the share of workers living in location j that commute to i to work is given by:

$$\pi_{i|j} = \frac{a_{ij} V_{ij}^{\theta}}{\sum_{i' \in \Omega} a_{i'j} V_{i'j}^{\theta}} = \frac{a_{ij} \left(\frac{w_i}{\kappa_{ij}}\right)^{\theta}}{\sum_{i' \in \Omega} a_{i'j} \left(\frac{w_{i'}}{\kappa_{i'j}}\right)^{\theta}}.$$

A.3.2 Rental Markets

Housing Demand Housing demand for residents in j commuting to i is given by

$$h_{ij} = (1-\alpha)\frac{w_i}{r_j}.$$

Let $\overline{w}_j = \sum_{i \in \Omega} \pi_{i|j} w_i$. Aggregating across working neighborhoods, we get the total housing demand, H_j^d:

$$H_j^d = R_j (1-\alpha)\frac{\overline{w}_j}{r_j}.$$

Housing Supply First-order condition for materials in the housing developer problem yields $r_j = \frac{\iota}{(1-\phi)G_j}\left(\frac{M_j}{L_j}\right)^{\phi}$. Using the zero profit condition and substituting for r_j gives us

$$\begin{aligned}
\frac{\iota}{(1-\phi)G_j}\left(\frac{M_j}{L_j}\right)^{\phi} G_j L_j^{\phi} M_j^{1-\phi} &= \iota M_j + p_j L_j \\
\frac{\iota}{(1-\phi)} M_j = \iota M_j + p_j L_j &\Rightarrow M_j = \frac{1-\phi}{\phi}\frac{p_j L_j}{\iota}
\end{aligned}$$

Again, from the zero profit condition,

$$\begin{aligned}
r_j &= \frac{\iota M_j + p_j L_j}{G_j L_j^{\phi} M_j^{1-\phi}} \\
&= \frac{\iota \frac{1-\phi}{\phi}\frac{p_j L_j}{\iota} + p_j L_j}{G_j L_j^{\phi}\left(\frac{1-\phi}{\phi}\frac{p_j L_j}{\iota}\right)^{1-\phi}} = \frac{\frac{1-\phi}{\phi} p_j + p_j}{G_j\left(\frac{1-\phi}{\phi}\frac{p_j}{\iota}\right)^{1-\phi}} \\
&= \underbrace{\frac{\iota^{1-\phi}}{G_j (1-\phi)^{1-\phi}\phi^{\phi}}}_{\equiv \rho_j} p_j^{\phi} = \rho_j p_j^{\phi}.
\end{aligned}$$

Using the land supply equation, we get the relationship between housing rent, housing demand and land

$$r_j = \rho_j \left(\frac{H_j}{L_j}\right)^{\psi},\ \psi \equiv \phi \times \overline{\psi}.$$

A.3.3 Welfare

Using similar arguments from the section on worker's location choice, the probability of a worker to work in neighborhood i and live in neighborhood j is $\exp\left(-v^{-\theta}\sum_{i,j\in\Omega}\psi_{ij}\right)$. Consequently, the expected utility of living in the MSA for such worker is:

$$E(V)=\int_0^\infty v\left(\sum_{i,j\in\Omega}\psi_{ij}\right)v^{-\theta-1}\exp\left(-v^{-\theta}\sum_{i,j\in\Omega}\psi_{ij}\right)dv.$$

Let $x=\left(\sum_{i,j\in\Omega}\psi_{ij}\right)v^{-\theta}$ so $x\in(\infty,0)$ for $v\in(0,\infty)$, $dx=-\left(\sum_{i,j\in\Omega}\psi_{ij}\right)\theta v^{-\theta-1}dv$ and $v=\left(\frac{x}{\sum_{i,j\in\Omega}\psi_{ij}}\right)^{-\frac{1}{\theta}}$. Then

$$\begin{aligned}
E(V)&=-\int_0^\infty v\exp\left(-v^{-\theta}\sum_{i,j\in\Omega}\psi_{ij}\right)\left[-\left(\sum_{i,j\in\Omega}\psi_{ij}\right)v^{-\theta-1}dv\right]\\
&=-\int_\infty^0\left(\frac{x}{\sum_{i,j\in\Omega}\psi_{ij}}\right)^{-\frac{1}{\theta}}\exp\left(-x\right)dx=\int_0^\infty\left(\frac{x}{\sum_{i,j\in\Omega}\psi_{ij}}\right)^{-\frac{1}{\theta}}\exp\left(-x\right)dx\\
&=\int_0^\infty x^{\left(1-\frac{1}{\theta}\right)-1}\exp\left(-x\right)dx\left(\sum_{i,j\in\Omega}\psi_{ij}\right)^{\frac{1}{\theta}}\\
&=\Gamma\left(1-\frac{1}{\theta}\right)\left(\sum_{i,j\in\Omega}a_{ij}V_{ij}^\theta\right)^{\frac{1}{\theta}}=\Gamma\left(1-\frac{1}{\theta}\right)\left(\sum_{i\in\Omega}a_{ij}\left(\frac{w_is_j}{\kappa_{ij}r_j^{1-\alpha}}\right)^\theta\right)^{\frac{1}{\theta}}.
\end{aligned}$$

A.4 Computational strategy

1. Guess array $\mathbb{V}^0$;
2. Compute $\lambda=\left[\sum_{i,j\in\Omega}a_{ij}V_{ij}^0\right]^{-1}$;
3. Calculate residents and workers by neighborhood using

$$R_j=\lambda\sum_{i\in\Omega}a_{ij}(V_{ij}^0)^\theta\overline{R}$$
$$n_i=\lambda\sum_{j\in\Omega}a_{ij}(V_{ij}^0)^\theta\overline{R}.$$

4. Derive neighborhood wages using

$$w_i=\beta\overline{A}_in_i^{\beta+\eta-1}.$$

5. Compute $\overline{w}$ and find housing by neighborhood using

$$H_j=\left[\frac{(1-\alpha)\overline{w}_j}{\rho_j}R_jL_j^\psi\right]^{\frac{1}{\psi}}.$$

6. Calculate rent using
$$r_j = \rho_j \left(\frac{H_j}{L_j}\right)^{\psi}.$$
7. Update indirect utility $\mathbb{V}^1$, where:
$$V_{ij}^1 = \frac{w_i s_j - r_j \overline{h}}{\kappa_{ij} r_j^{1-\alpha}}$$
8. Check if $||\mathbb{V}^1 - \mathbb{V}^0|| < 10^{-6}$. Stop if true. If not, set $\mathbb{V}^0_{new} = .25\mathbb{V}^1 + .75\mathbb{V}^0_{old}$.

Appendix B

Appendix to Chapter 2

B.1 Proofs

Proof of Proposition 1

We begin by first proving the following Lemma:

Lemma 4 *Given sectoral prices and wages, the firm i's optimality condition for output, intermediate goods and labor satisfies*

$$(1-\alpha_i)\omega_{ij}p_iy_i = \frac{R_i}{R_j}p_jx_{ij} \quad \textit{for all } j, \textit{ and} \tag{B.1}$$

$$\alpha_ip_iy_i = R_iw\ell_i. \tag{B.2}$$

In addition, given sectoral prices and wages, the firm i's cost function for a given output level is given by

$$C(y_i; w, \{p_{ij}\}) = \Theta_i^{-1}\frac{y_i}{z_i}w^{\alpha_i}R_i^{\alpha_i}\prod_{j=1}^{N}\left(p_j\frac{R_i}{R_j}\right)^{\omega_{ij}(1-\alpha_i)}, \tag{B.3}$$

where $\Theta_i \equiv \alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}\prod_{j=1}^{N}\omega_{ij}^{\omega_{ij}(1-\alpha_i)}$.

Proof Because the production constraint binds, we can rewrite the maximization problem as

$$\max_{\ell_i,\{x_{ij}\}_{j=1}^N} \quad p_iz_i\ell_i^{\alpha_i}\left(\prod_{j=1}^{N}x_{ij}^{\omega_{ij}}\right)^{1-\alpha_i} - R_i\left\{\sum_j p_{ij}x_{ij} + w\ell_i\right\}$$

Optimality conditions are

$$(1-\alpha_i)\omega_{ij}p_iy_i = R_ip_{ij}x_{ij}$$
$$\alpha_ip_iy_i = R_iw\ell_i.$$

In order for it not to be profitable for the firm to focus on selling their goods either to firms or to households, the marginal revenues have to be equal: $p_i = R_i p_{ji}$. Thus, we can rewrite the optimality condition for intermediate goods as

$$(1-\alpha_i)\omega_{ij}p_i y_i = \frac{R_i}{R_j} p_j x_{ij}.$$

The cost minimization problem for the firm is

$$\min_{\ell_i, \{x_{ij}\}} R_i \left[\sum_j p_{ij}x_{ij} + w\ell_i\right] \quad \text{s.t.} \quad y_i = z_i \ell_i^{\alpha_i}\left(\prod_{j=1}^{N} x_{ij}^{\omega_{ij}}\right)^{1-\alpha_i}.$$

First-order conditions imply the following ratios:

$$\frac{p_{ij}}{w} = \frac{(1-\alpha_i)\omega_{ij}}{\alpha_i}\frac{\ell_i}{x_{ij}} \quad \text{and} \quad \frac{p_{ij}}{p_{ik}} = \frac{\omega_{ij}}{\omega_{ik}}\frac{x_{ik}}{x_{ij}},\ j \neq k.$$

Therefore, we can write

$$\prod_j x_{ij}^{\omega_{ij}} = w\ell_i \left(\frac{1-\alpha_i}{\alpha_i}\right) \prod_j \left(\frac{\omega_{ij}}{p_{ij}}\right)^{\omega_{ij}},$$

$$\prod_{k\neq j} x_{ik}^{\omega_{ik}} = \left(\frac{p_{ij}}{\omega_{ij}}x_{ij}\right)^{1-\omega_{ij}} \prod_{k\neq j}\left(\frac{\omega_{ik}}{p_{ik}}\right)^{\omega_{ik}}$$

Plugging these results back in the production function, we get

$$y_i = z_i\ell_i\left[w\left(\frac{1-\alpha_i}{\alpha_i}\right)\right]^{1-\alpha_i}\prod_j\left(\frac{\omega_{ij}}{p_{ij}}\right)^{\omega_{ij}(1-\alpha_i)} \tag{ℓ_i}$$

$$y_i = z_i\left[\left(\frac{p_{ij}}{w}\right)\frac{\alpha_i}{\omega_{ij}(1-\alpha_i)}x_{ij}\right]^{\alpha_i}\left[x_{ij}\left(\frac{p_{ij}}{\omega_{ij}}\right)^{1-\omega_{ij}}\prod_{k\neq j}\left(\frac{\omega_{ik}}{p_{ik}}\right)^{\omega_{ik}}\right]^{1-\alpha_i} \tag{x_{ij}}$$

$$= z_i x_{ij}\left(\frac{p_{ij}}{\omega_{ij}}\right)\left[\frac{\alpha_i}{w(1-\alpha_i)}\right]^{\alpha_i}\prod_{k=1}^{N}\left(\frac{\omega_{ik}}{p_{ik}}\right)^{\omega_{ik}(1-\alpha_i)}$$

We then get the following conditional input demands:

$$\ell_i = \frac{y_i}{z_i}\left[w\left(\frac{1-\alpha_i}{\alpha_i}\right)\right]^{\alpha_i-1}\prod_j\left(\frac{\omega_{ij}}{p_{ij}}\right)^{-\omega_{ij}(1-\alpha_i)},$$

$$x_{ij} = \frac{y_i}{z_i}\left(\frac{\omega_{ij}}{p_{ij}}\right)\left[\frac{\alpha_i}{w(1-\alpha_i)}\right]^{-\alpha_i}\prod_{k=1}^{N}\left(\frac{\omega_{ik}}{p_{ik}}\right)^{-\omega_{ik}(1-\alpha_i)}.$$

Therefore, the cost function is given by

$$
\begin{aligned}
R_i\left[\sum_j p_{ij}x_{ij}+w\ell_i\right] &= R_i\frac{y_i}{z_i}\prod_{j=1}^{N}\left(\frac{\omega_{ij}}{p_{ij}}\right)^{-\omega_{ij}(1-\alpha_i)}\left[\sum_j\omega_{ij}\left[\frac{\alpha_i}{w(1-\alpha_i)}\right]^{-\alpha_i}+w\left[w\left(\frac{1-\alpha_i}{\alpha_i}\right)\right]^{\alpha_i-1}\right]\\
&= R_i\frac{y_i}{z_i}\prod_{j=1}^{N}\left(\frac{\omega_{ij}}{p_{ij}}\right)^{-\omega_{ij}(1-\alpha_i)}\left\{\left[\frac{\alpha_i}{w(1-\alpha_i)}\right]^{-\alpha_i}+w\left[w\left(\frac{1-\alpha_i}{\alpha_i}\right)\right]^{\alpha_i-1}\right\}\\
&= R_i\frac{y_i}{z_i}w^{\alpha_i}\prod_{j=1}^{N}\left(\frac{\omega_{ij}}{p_{ij}}\right)^{-\omega_{ij}(1-\alpha_i)}\left[\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\alpha_i}+\left(\frac{\alpha_i}{1-\alpha_i}\right)^{1-\alpha_i}\right]\\
&= R_i\frac{y_i}{z_i}w^{\alpha_i}\prod_{j=1}^{N}\left(\frac{\omega_{ij}}{p_j}R_j\right)^{-\omega_{ij}(1-\alpha_i)}\left[\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}\right]^{-1}\\
&= \frac{y_i}{z_i}w^{\alpha_i}R_i^{\alpha_i+\sum_j\omega_{ij}(1-\alpha_i)}\prod_{j=1}^{N}\omega_{ij}^{-\omega_{ij}(1-\alpha_i)}\prod_{j=1}^{N}\left(\frac{p_j}{R_j}\right)^{\omega_{ij}(1-\alpha_i)}\left[\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}\right]^{-1}\\
&= \Theta_i^{-1}\frac{y_i}{z_i}w^{\alpha_i}R_i^{\alpha_i}\prod_{j=1}^{N}\left(p_j\frac{R_i}{R_j}\right)^{\omega_{ij}(1-\alpha_i)},
\end{aligned}
$$

where $\Theta_i \equiv \alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}\prod_{j=1}^{N}\omega_{ij}^{\omega_{ij}(1-\alpha_i)}$. ■

Normalize $P \equiv \Pi_i p_i^{\nu_i} = 1$. Let $g_i \equiv p_i y_i$. Then, using (B.1),

$$
\begin{aligned}
p_i y_i &= p_i c_i + \sum_j p_i x_{ji}\\
&= p_i c_i + \sum_j (1-\alpha_j)\omega_{ji}p_j y_j\frac{R_i}{R_j}.
\end{aligned}
$$

Stacking the equations, we have

$$
\begin{bmatrix} g_1\\ \vdots\\ g_N\end{bmatrix} = \begin{bmatrix}\nu_1\\ \vdots\\ \nu_N\end{bmatrix}c+\left(\begin{bmatrix}(1-\alpha_1)\omega_{11} & \dots & (1-\alpha_1)\omega_{N1}\\ \vdots & \ddots & \vdots\\ (1-\alpha_N)\omega_{1N} & \dots & (1-\alpha_N)\omega_{NN}\end{bmatrix}\circ\begin{bmatrix}\frac{R_1}{R_1} & \dots & \frac{R_1}{R_N}\\ \vdots & \ddots & \vdots\\ \frac{R_N}{R_1} & \dots & \frac{R_N}{R_N}\end{bmatrix}\right)\begin{bmatrix} g_1\\ \vdots\\ g_N\end{bmatrix}
$$

where $\circ$ denote the Hadamard (entrywise) product. Normalizing $P = 1$ implies $p_i c_i = \nu_i c$. Define

$$
\Omega \equiv \begin{bmatrix}(1-\alpha_1)\omega_{11} & (1-\alpha_1)\omega_{12} & \dots & (1-\alpha_1)\omega_{1N}\\ \vdots & \vdots & \ddots & \vdots\\ (1-\alpha_N)\omega_{N1} & (1-\alpha_N)\omega_{N2} & \dots & (1-\alpha_N)\omega_{NN}\end{bmatrix},\quad \Lambda \equiv \begin{bmatrix}\frac{R_1}{R_1} & \frac{R_1}{R_2} & \dots & \frac{R_1}{R_N}\\ \vdots & \vdots & \ddots & \vdots\\ \frac{R_N}{R_1} & \frac{R_N}{R_2} & \dots & \frac{R_N}{R_N}\end{bmatrix}.
$$

Let $\mathbf{g} \equiv [g_1 \dots g_N]'$ and $\boldsymbol{\nu} \equiv [\nu_1 \dots \nu_N]'$. Then, from the algebra above:

$$
\begin{aligned}
\mathbf{g} &= \boldsymbol{\nu}c + (\Omega'\circ\Lambda)\mathbf{g}\\
\Rightarrow \mathbf{g} &= \left(I-\Omega'\circ\Lambda\right)^{-1}\boldsymbol{\nu}c.
\end{aligned}
$$

Using (B.3), perfect competition implies that sectoral price equals marginal cost:

$$p_i = \Theta_i^{-1} \frac{1}{z_i} w^{\alpha_i} R_i^{\alpha_i} \prod_{j=1}^{N} \left(p_j \frac{R_i}{R_j} \right)^{\omega_{ij}(1-\alpha_i)}$$

and thus

$$w^{\alpha_i} = \Theta_i p_i \prod_{j=1}^{N} (p_j)^{-\omega_{ij}(1-\alpha_i)} z_i R_i^{-\alpha_i} \prod_{j=1}^{N} \left(\frac{R_i}{R_j} \right)^{-\omega_{ij}(1-\alpha_i)}.$$

Define $\beta \equiv (I - \Omega')^{-1} \boldsymbol{\nu}$, which is a centrality measure. It can be shown that $\sum_i \alpha_i \beta_i = 1$. From the definition of β, $-\Omega'\beta = \boldsymbol{\nu} - \beta$. Therefore,

$$\prod_i p_i^{\beta_i} \prod_j \left(p_j^{-\omega_{ij}(1-\alpha_i)\beta_i} \right) = \prod_i p_i^{\beta_i - \sum_j \omega_{ji}(1-\alpha_j)\beta_j} = \prod_i p_i^{\beta_i + \nu_i - \beta_i} = \prod_i p_i^{\nu_i} = P = 1.$$

We can write the wage in the economy as a function of labor shares, $\{\alpha_i\}$, intermediate good shares, $\{\omega_{ij}\}$, and the interest rates faced by the firms, $\{R_i\}$:

$$\begin{aligned}
\prod_i w^{\alpha_i \beta_i} &= \prod_i \Theta_i^{\beta_i} p_i^{\beta_i} \prod_{j=1}^{N} (p_j)^{-\omega_{ij}(1-\alpha_i)\beta_i} z_i^{\beta_i} R_i^{-\alpha_i \beta_i} \prod_{j=1}^{N} \left(\frac{R_i}{R_j} \right)^{-\omega_{ij}(1-\alpha_i)\beta_i} \\
\Rightarrow w &= \prod_i \Theta_i^{\beta_i} z_i^{\beta_i} R_i^{-\alpha_i \beta_i} \prod_{j=1}^{N} \left(\frac{R_i}{R_j} \right)^{-\omega_{ij}(1-\alpha_i)\beta_i}.
\end{aligned}$$

Let $\mathbf{s} \equiv [s_1 \ldots s_N]'$ and $\mathbf{R} \equiv [R_1 \ldots R_N]'$. Denote $\oslash$ as the Hadamard division and $\mathbf{1}$ as a vector of ones. Using (B.1) and (B.2) on the government's budget constraint:

$$\begin{aligned}
\tau w \ell &= \sum_i^N s_i \left[\sum_j (p_{ij} x_{ij} - p_{ji} x_{ji}) + w \ell_i \right] \\
&= \sum_i^N s_i \left(\sum_j p_{ij} x_{ij} + w \ell_i \right) - \sum_i^N s_i \left(\sum_j p_{ji} x_{ji} \right) \\
&= \sum_i^N s_i \left(\sum_j \frac{(1-\alpha_i)\omega_{ij} p_i y_i}{R_i} + \frac{\alpha_i p_i y_i}{R_i} \right) - \sum_i^N s_i \left(\sum_j \frac{(1-\alpha_j)\omega_{ji} p_j y_j}{R_j} \right) \\
&= \sum_i^N s_i \left((1-\alpha_i) \sum_j \omega_{ij} + \alpha_i \right) \frac{g_i}{R_i} - \sum_i^N s_i \left(\sum_j \frac{(1-\alpha_j)\omega_{ji} g_j}{R_j} \right) \\
&= \sum_i^N s_i \frac{g_i}{R_i} - \sum_i^N s_i \left(\sum_j \frac{(1-\alpha_j)\omega_{ji} g_j}{R_j} \right) \\
&= ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))')\mathbf{g}.
\end{aligned}$$

Using it in the household's budget constraint,

$$\begin{aligned}
c &= (1-\tau)w\ell \\
&= w\ell - ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))')\mathbf{g} \\
&= w\ell - ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))') \left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu} c.
\end{aligned}$$

Let $\mathbf{z} \equiv [z_1 \ \dots \ z_N]'$. Through market clearing, we can express GDP as a function of total labor supply, ℓ, sectoral productivities $\{z_i\}$, sectoral interest rates $\{R_i\}$, sectoral subsidies $\{s_i\}$, and exogenous parameters:

$$GDP(\mathbf{z}, \mathbf{R}, \mathbf{s}) = \frac{\prod_i \Theta_i^{\beta_i} z_i^{\beta_i} R_i^{-\alpha_i \beta_i} \prod_{j=1}^N \left(\frac{R_i}{R_j}\right)^{-\omega_{ij}(1-\alpha_i)\beta_i}}{1 + ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))') \left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu}} \ell.$$

Using this result in the household's budget constraint, we can write the tax rate τ as

$$\tau = \frac{((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))') \left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu}}{1 + ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))') \left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu}}.$$

From the sector's zero-profit condition, we take logs:

$$\begin{aligned}
\log p_i &= -(\log \Theta_i + \log z_i) + \alpha_i(\log w + \log R_i) + \sum_{j=1}^N (1-\alpha_i)\omega_{ij} \left(\log p_j + \log R_i - \log R_j\right) \\
&= -(\log \Theta_i + \log z_i) + \log R_i + \alpha_i \log w + \sum_{j=1}^N (1-\alpha_i)\omega_{ij} \left(\log p_j - \log R_j\right)
\end{aligned}$$

Let $\mathbf{p} \equiv [p_1 \ \dots \ p_N]'$, $\Theta \equiv [\Theta_1 \ \dots \ \Theta_N]'$ and $\boldsymbol{\alpha} \equiv [\alpha_1 \ \dots \ \alpha_N]'$. Then, we can write the system as

$$\log \mathbf{p} = -(\log \Theta + \log \mathbf{z}) + \log \mathbf{R} + \boldsymbol{\alpha} \log w + \Omega(\log \mathbf{p} - \log \mathbf{R}).$$

Which yields

$$\log \mathbf{p} = \log \mathbf{R} + (I - \Omega)^{-1} \left[\boldsymbol{\alpha} \log w - (\log \Theta + \log \mathbf{z})\right].$$

Taking logs on the equation for wages

$$\begin{aligned}
\log w &= \sum_i \beta_i \left[\log \Theta_i + \log z_i - \alpha_i \log R_i - (1-\alpha_i) \sum_{j=1}^N \omega_{ij} \left(\log R_i - \log R_j\right)\right] \\
&= \sum_i \beta_i \left[\log \Theta_i + \log z_i - \log R_i + (1-\alpha_i) \sum_{j=1}^N \omega_{ij} \log R_j\right] \\
&= \beta' \left[\log \Theta + \log \mathbf{z} - (I - \Omega) \log \mathbf{R}\right]
\end{aligned}$$

We can then solve for the price vector:

$$\begin{aligned}\log \mathbf{p} &= \log \mathbf{R} + (I-\Omega)^{-1}\left\{\boldsymbol{\alpha}\beta'\left[\log\Theta + \log \mathbf{z} - (I-\Omega)\mathbf{R}\right] - (\log\Theta + \log\mathbf{z})\right\} \\ &= \log\mathbf{R} - (I-\Omega)^{-1}(I-\boldsymbol{\alpha}\beta')(\log\Theta + \log\mathbf{z}) - (I-\Omega)^{-1}\boldsymbol{\alpha}\beta'(I-\Omega)\mathbf{R} \\ &= \left[I - (I-\Omega)^{-1}\boldsymbol{\alpha}\beta'(I-\Omega)\right]\log\mathbf{R} - (I-\Omega)^{-1}(I-\boldsymbol{\alpha}\beta')(\log\Theta+\log\mathbf{z})\end{aligned}$$

Proof of Proposition 2

Household's optimality condition is given by

$$\frac{\ell^{\epsilon}}{c^{\sigma}} = (1-\tau)w.$$

The labor wedge is implicitly defined by the relation

$$\frac{\ell^{\epsilon}}{c^{\sigma}} = (1-\tau_\ell)\frac{c}{\ell}.$$

Combining both, we get

$$(1-\tau_\ell)\frac{c}{\ell} = (1-\tau)w.$$

The expression $1-\tau$ can be written as

$$1-\tau = \frac{1}{1+\left((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))'\right)(I-\Omega'\circ\Lambda)^{-1}\boldsymbol{\nu}}.$$

Using the results from Proposition 1, the equivalence results becomes

$$\frac{1-\tau_\ell}{1+\left((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))'\right)(I-\Omega'\circ\Lambda)^{-1}\boldsymbol{\nu}} = \frac{1}{1+\left((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))'\right)(I-\Omega'\circ\Lambda)^{-1}\boldsymbol{\nu}}.$$

which implies that $1-\tau_\ell = 1$ and thus $\tau_\ell = 0$.

Proof of Lemma 1

Starting from the labor wedge definition, we get

$$\ell^{1+\varphi} = (1-\tau_\ell(\mathbf{s},\mathbf{R}))c^{1-\sigma}.$$

Given that $c(\mathbf{z},\mathbf{R},\mathbf{s}) = A(\mathbf{z})\mu(\mathbf{R},\mathbf{s})\ell$, we take logs

$$(1+\varphi)\log\ell = (1-\sigma)(\log A(\mathbf{z}) + \log\mu(\mathbf{R},\mathbf{s}) + \log\ell) + \log(1-\tau_\ell(\mathbf{s},\mathbf{R})),$$

which gives us

$$\log\ell = \frac{1-\sigma}{\sigma+\varphi}(\log A(\mathbf{z}) + \log\mu(\mathbf{R},\mathbf{s})) + \frac{1}{\sigma+\varphi}\log(1-\tau_\ell(\mathbf{s},\mathbf{R})),$$

and thus

$$\begin{aligned}\log c &= \log A(\mathbf{z}) + \log\mu(\mathbf{R},\mathbf{s})\frac{1-\sigma}{\sigma+\varphi}(\log A(\mathbf{z}) + \log\mu(\mathbf{R},\mathbf{s})) + \frac{1}{\sigma+\varphi}\log(1-\tau_\ell(\mathbf{s},\mathbf{R})) \\ &= \frac{1+\varphi}{\sigma+\varphi}(\log A(\mathbf{z}) + \log\mu(\mathbf{R},\mathbf{s})) + \frac{1}{\sigma+\varphi}\log(1-\tau_\ell(\mathbf{s},\mathbf{R})).\end{aligned}$$

B.2 Deriving the lump-sum transfers to consumers

If the idiosyncratic borrowing cost e_i is transfered lump-sum to the consumer instead of being thrown into the ocean, the transfer, Ψ, is given by

$$\Psi = \sum_{i=1}^{N} e_i \left[\sum_{j=1}^{N} (p_{ij}x_{ij} - p_{ji}x_{ji}) + w\ell_i \right].$$

Remember that $e_i = R_i + s_i - 1$, since we set $R = 1$. Thus

$$\begin{aligned}
\Psi &= \sum_{i}^{N} (R_i + s_i - 1) \left[\sum_{j} (p_{ij}x_{ij} - p_{ji}x_{ji}) + w\ell_i \right] \\
&= \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} p_{ij}x_{ij} + w\ell_i \right) - \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} p_{ji}x_{ji} \right) \\
&= \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} \frac{(1-\alpha_i)\omega_{ij}p_i y_i}{R_i} + \frac{\alpha_i p_i y_i}{R_i} \right) - \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} \frac{(1-\alpha_j)\omega_{ji}p_j y_j}{R_j} \right) \\
&= \sum_{i}^{N} (R_i + s_i - 1) \left((1-\alpha_i) \sum_{j} \omega_{ij} + \alpha_i \right) \frac{g_i}{R_i} - \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} \frac{(1-\alpha_j)\omega_{ji} g_j}{R_j} \right) \\
&= \sum_{i}^{N} (R_i + s_i - 1) \frac{g_i}{R_i} - \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} \frac{(1-\alpha_j)\omega_{ji} g_j}{R_j} \right) \\
&= [((\mathbf{R} + \mathbf{s} - \mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R} + \mathbf{s} - \mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))']\mathbf{g},
\end{aligned}$$

where $\mathbf{1}$ is a vector where all entries equal one. Using it in the household's budget constraint,

$$\begin{aligned}
c &= (1-\tau)w\ell + \Psi \\
&= w\ell - ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))')\mathbf{g} + [((\mathbf{R} + \mathbf{s} - \mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R} + \mathbf{s} - \mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))']\mathbf{g} \\
&= w\ell + [((\mathbf{R} - \mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R} - \mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))']\mathbf{g} \\
&= w\ell + [((\mathbf{R} - \mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R} - \mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'] \left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu} c.
\end{aligned}$$

The expression for GDP is as follows

$$GDP(\mathbf{z}, \mathbf{R}, \mathbf{s}) = \frac{\prod_i \Theta_i^{\beta_i} z_i^{\beta_i} R_i^{-\alpha_i \beta_i} \prod_{j=1}^{N} \left(\frac{R_i}{R_j}\right)^{-\omega_{ij}(1-\alpha_i)\beta_i}}{1 - [((\mathbf{R} - \mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R} - \mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'] \left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu}} \ell.$$

Using this result in the household's budget constraint, we can write the tax rate τ as

$$\tau = \frac{((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))') \left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu}}{1 - [((\mathbf{R} - \mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R} - \mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'] \left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu}}.$$

In this case, the efficiency wedge is given by

$$\mu(\mathbf{R},\mathbf{s}) \equiv \frac{\prod_i R_i^{-\alpha_i\beta_i} \prod_{j=1}^{N} \left(\frac{R_i}{R_j}\right)^{-\omega_{ij}(1-\alpha_i)\beta_i}}{1 - \left[((\mathbf{R}-\mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R}-\mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right] (I - \Omega' \circ \Lambda)^{-1} \boldsymbol{\nu}}.$$

The labor wedge can be derived from

$$(1-\tau_\ell)\frac{c}{\ell} = (1-\tau)w$$

$$(1-\tau_\ell) = 1 - \left[((\mathbf{R}+\mathbf{s}-\mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R}+\mathbf{s}-\mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right] \left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu}.$$